# THE ROTATING DISK IN ODTOE: AN ANGULAR TRANSPOSITION OF THE PROJECTIVE LIGHT POLE ON THE $\nu_\Phi$ SPECTRUM

(Вращающийся диск в ODTOE: угловая транспозиция проективного полюса света на спектре $\nu_\Phi$)

*Apparent stasis as one affine chart of the $\Phi$-iteration spectrum and the $(\pi-3)^2$ residue of the observed turn*

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## ABSTRACT

Within the Observer-Dependent Theory of Everything (ODTOE) [1], the rotating disk is treated as an angular transposition of the projective result on the intrinsic rest frame of light [2]. Observing the disk at a single sampling frequency $\nu_{\text{obs}}$ is the selection of one affine chart of the $\Phi$-iteration spectrum $\nu_\Phi$ through a rank-limited operator $\hat{O}_B$ with a single projector $P_A$. The apparent stasis (strobe matching, $\nu_\Phi \to 0$) and the apparent omnipresence (the limit $\nu_\Phi \to \infty$) are two antipodal charts $\chi_0$, $\chi_\infty$, identified by the Möbius inversion $\iota_M$ into a single projective point $[0:\infty] \in \mathbb{R}P^1$. This thesis is formalized by Proposition 1 (the projective pole of rotation). A complete $2\pi$ turn through a rank-limited chart carries the dimensionless spiral residue $(\pi-3)^2 \approx 0.0200$ (Corollary 1), coinciding with the corpus spiral gap. Apparent stasis is one affine chart of the $\nu_\Phi$ spectrum; angular momentum $L$, the Sagnac effect, and Lense–Thirring dragging are invariants of the complete basis. Motion is chart-dependent at level L1 and invariant-real at levels L2/L3. Falsifiers are stated: the numerical $(\pi-3)^2$ residue to 50 decimal places, and the decisive criterion that the Sagnac phase and the angular momentum $L$ are independent of the observer's sampling frequency.

**Keywords:** ODTOE, rotating disk, projective geometry, $\Phi$-iteration spectrum, Möbius inversion, $\mathbb{R}P^1$, Sagnac effect, Lense–Thirring dragging, Ehrenfest paradox, spiral gap $(\pi-3)^2$.

# I. INTRODUCTION

A rotating disk observed at a single sampling frequency poses a vivid question about the status of motion. When the strobe frequency matches the angular speed, the disk appears static; at nearby but unequal frequencies the wagon-wheel illusion arises, an apparent counter-rotation. Yet the same disk carries a definite angular momentum whose reality is confirmed by the gyroscopic effect and by the Sagnac effect. The question follows: are the “static” and the “rotating” readings two incompatible descriptions, or two projections of one object?

The Observer-Dependent Theory of Everything (ODTOE) [1] supplies an operator-algebraic mechanism for the answer. The central object is the self-observation map $\Phi = \iota \circ \hat{O}$ acting on the Hilbert space of potential states; the spectrum of its iteration frequencies $\nu_\Phi$ is a structural object amenable to projective geometry. The work on the intrinsic rest frame of light [2] establishes that on this spectrum the points $\nu_\Phi = 0$ and $\nu_\Phi = \infty$ coincide as a single projective point $[0 : \infty] \in \mathbb{R}P^1$, glued by the antipodal Möbius inversion. The present work performs an angular transposition of that result: what [2] does for the *frequency* axis of the spectrum is carried over to the *rotational* axis. Observation at a single frequency $\nu_{\text{obs}}$ is the selection of one affine chart of the $\nu_\Phi$ spectrum (rank-limited $\hat{O}_B$, a single projector $P_A$); the apparent stasis and the apparent omnipresence are two antipodal charts of that spectrum. The thesis: apparent rotation is an artifact of the single-frequency chart, and observed motion is a property of the pair (observer frequency + disk).

The contribution of the work has three dimensions. **(a) Geometric:** the angular projective gluing of the charts $\chi_0$ and $\chi_\infty$ on $\mathbb{R}P^1$ through the same Möbius inversion $\iota_M$ as in [2]. **(b) Logical:** apparent stasis and apparent rotation are artifacts of the choice of affine chart on one projective point. **(c) Epistemic:** observed rotation is a property of the pair $(\nu_{\text{obs}}, \text{disk})$; the angular momentum $L$, the Sagnac phase, and Lense–Thirring dragging are observer-invariant quantities that survive the complete basis.

Structure of the article: Section II is the literature review and the placement of the work within the ODTOE corpus; Section III recapitulates the $\Phi/\nu_\Phi$ formalism (cited, without re-derivation); Section IV is the NEW material, Proposition 1 and the four formula labels of the angular chart; Section V treats the invariants of the complete basis and Corollary 1 with the $(\pi-3)^2$ residue; Section VI states the falsifiers; Section VII the limitations.

# II. LITERATURE REVIEW AND POSITION WITHIN THE ODTOE CORPUS

The paradox of the rigid rotating disk goes back to Ehrenfest [3]: under relativistic rotation, the circumference and the radius behave incompatibly with the Euclidean geometry of the disk at rest. The Sagnac effect [4] established that in a rotating frame counter-propagating light beams accumulate a measurable phase difference proportional to the angular speed and the enclosed area; this effect underlies ring laser gyroscopes. The general-relativistic dragging of inertial frames by a rotating mass was described by Lense and Thirring [5]. From the side of information theory, the frequency limit of resolvability is set by the Shannon–Nyquist sampling theorem [6]: observation at a finite sampling frequency does not resolve structure above half the sampling rate. The wagon-wheel perceptual illusion, an apparent counter-rotation under strobed illumination or discrete frame sampling, has been studied experimentally by Purves and co-authors [7].

Position of the work within the ODTOE corpus. The present work is an angular transposition of the frequency result of the work on the intrinsic rest frame of light [2]: the same projective pole $[0 : \infty]$ and the same Möbius inversion are carried from the frequency axis to the rotational axis. It reuses the unified operator $\hat{O}_B$ and the Banach contraction [8], the quaternion realization of orientation $R = q \cdot \Psi \cdot \bar{q}$ for angular momentum [9], the reading of motion as $\Phi$-iteration and the tact counter [10], the dimensionality scaffold of the observer and the octaves of reality [11], and the dimensionless spiral gap $(\pi - 3)^2$ from the work on the number $\pi$ [12]. The geometry of the projective line $\mathbb{R}P^1$ and the role of projective methods in the foundations of physics are systematized by Penrose [13].

A systematic review revealed no prior work gluing the apparent-stasis reading and the apparent-rotation reading of a disk as one projective point on $\mathbb{R}P^1$. Each separate axis (the Ehrenfest paradox, the Sagnac effect, Lense–Thirring dragging, the Nyquist sampling limit, the wagon-wheel illusion) has a rich prehistory; the novelty is their conjunction in the projective picture of the $\nu_\Phi$ spectrum.

## III. THE ODTOE $\Phi/\nu_\Phi$ FORMALISM: A RECAPITULATION

This section reproduces the corpus formulas needed below. All formulas are cited verbatim, without re-derivation; the original material (Proposition 1, Corollary 1) appears in §IV–§V. The epistemic status of the section's formulas is inherited structural identities (level L2).

Axiom A fixes the basic relation between potential and observed reality [1]: the observed configuration $R = \hat{O}(\Psi)$ is the image of the potential field $\Psi \in \mathcal{H}$ under the observation operator. The composition of the immersion operator and the observation operator gives the self-observation map (the strange loop) $\Phi = \iota \circ \hat{O} : \mathcal{H} \to \mathcal{H}$ [1].

The unified observation operator $\hat{O}_B$ and the Banach contraction are cited from the work on the unified operator [8]. The observation operator depends on the coherence $B$ and is realized through a rank-limited projection:

$$\hat{O}_B(\Psi) = B \cdot P_A(\Psi) + (1 - B)\,\eta_B(\Psi), \qquad \Phi_{B,S} = \iota_S \circ \hat{O}_B. \tag{1}$$

The existence and uniqueness of the fixed point $\Psi^* = \Phi(\Psi^*)$ is supplied by the Banach theorem: the operator $\Phi_{B,S}$ is a contraction with constant

$$q = B \cdot S + (1 - B)\sqrt{1 - S^2} < 1, \qquad (B, S) \in (0, 1)^2. \tag{2}$$

In the inequality (2), $P_A$ is the projector onto the subspace resolved by the observer;

the rank of $\hat{O}_B$ is limited by the number of projectors held simultaneously. A single sampling frequency corresponds to a single projector $P_A$ — this fact carries the load of §IV.

Motion and time in ODTOE are read as $\Phi$-iteration [10]: time is the counter of iterations $n$ of the self-observation map, and each $\Phi$ cycle produces one discrete step. The duration of one iteration in the observer's own frame is denoted $\tau_{\text{step}}$, and the tact frequency of $\Phi$-iterations is defined as $\nu_\Phi \equiv 1/\tau_{\text{step}}$ [2]. On the $\nu_\Phi$ spectrum the work [2] establishes Theorem 1: the points $\nu_\Phi = 0$ and $\nu_\Phi = \infty$ coincide as a single projective point $[0 : \infty] \in \mathbb{R}P^1$, glued by the antipodal Möbius inversion $\iota_M : [a : b] \mapsto [b : a]$. These results are inherited by the present work (T2, level L2) and are carried from the frequency axis to the rotational axis in §IV.

# IV. THE PROJECTIVE POLE OF ROTATION: PROPOSITION 1

This section, together with Corollary 1 of §V, constitutes the sole NEW material of the work. The angular transposition carries the projective construction of [2] from the frequency axis of the $\nu_\Phi$ spectrum to the rotational axis: observation of the disk at a single frequency $\nu_{\text{obs}}$ is the selection of one affine chart, and the apparent limits "stasis" and "omnipresence" are two antipodal charts glued by the same Möbius inversion.

## IV.1. The single-frequency angular chart

Let an observer with a single sampling frequency $\nu_{\text{obs}}$ register a disk rotating at angular speed $\omega$ through a rank-limited operator $\hat{O}_B$ with a single projector $P_A$ (1). A single sampling frequency selects one affine chart $\chi_\nu$ of the angular $\nu_\Phi$ spectrum:

$$\chi_\nu : \nu_{\text{obs}} \longmapsto \big(\nu_\Phi \text{ chart realized by one projector } P_A\big), \qquad \text{rank}\, \hat{O}_B = 1. \tag{3}$$

The chart $\chi_\nu$ (3) is the angular analogue of the affine chart of the frequency spectrum in [2]: the rank-limitation of $\hat{O}_B$ to a single projector $P_A$ means that the observer holds one angular frequency of resolution per tact. Epistemic status: dimensionless projective structure, an observer-invariant point (level L2).

## IV.2. The angular antipodal gluing

In the limit of strobe matching, when the sampling frequency is a multiple of $\omega$, successive frames are indistinguishable, and the chart reads as $\nu_\Phi \to 0$ ("the disk is static"). In the limit $\nu_\Phi \to \infty$ the chart reads as "the disk is everywhere at once" — the absence of a distinguished angular position per tact. These two limits map to the homogeneous coordinates $[1 : 0]$ and $[0 : 1]$ and are identified by the same antipodal Möbius inversion as in the work [2]:

$$\iota_M : \mathbb{R}P^1 \to \mathbb{R}P^1, \quad [a:b] \mapsto [b:a]; \qquad \chi_0 \leftrightarrow [1:0], \quad \chi_\infty \leftrightarrow [0:1] \implies [0:\infty] \in \mathbb{R}P^1. \tag{4}$$

The points $[1 : 0]$ and $[0 : 1]$ form an orbit of the inversion $\iota_M$ of length 2 and are identified as a single projective point $[0 : \infty]$. This is the projective identity of the apparent stasis and the apparent omnipresence of the disk on $\mathbb{R}P^1$. The difference between the charts $\chi_0$ and $\chi_\infty$ is a property of the pair $(\nu_{\text{obs}}, \text{disk})$; the projective point $[0 : \infty]$ is structural. The inversion $\iota_M$ (4) is the same operation as in [2], a single shared construction.

## IV.3. The observed angular phase

The observed angular phase per sample is a function of the chosen chart — the rotation accumulated between two successive samples:

$$\varphi_{\text{obs}}(\nu) = \omega \cdot \tau_{\text{step}} = \frac{\omega}{\nu_\Phi}, \tag{5}$$

where $\tau_{\text{step}} = 1/\nu_\Phi$ is the duration of one $\Phi$-tact [10]. The quantity $\varphi_{\text{obs}}(\nu)$ (5) is the chart-dependent reading: under strobe matching $\varphi_{\text{obs}} \to 0 \pmod{2\pi}$ (chart $\chi_0$), and as $\nu_\Phi \to \infty$ the value $\varphi_{\text{obs}} \to 0$ as the limit of infinitely frequent sampling (chart $\chi_\infty$). Both limits are antipodes on $\mathbb{R}P^1$, glued through (4). Epistemic status: dimensionless projective structure (level L2).

## IV.4. The residue of the observed turn

Upon a complete $2\pi$ turn through the rank-limited chart $\chi_\nu$, the observed angular measure carries a spiral residue — the structural discrepancy between the continuous $2\pi$ turn and its discretely-observed reconstruction:

$$\Delta_{\text{spiral}} = (\pi - 3)^2, \tag{6}$$

defined here as a dimensionless quantity and evaluated in Corollary 1 of §V. The residue $\Delta_{\text{spiral}}$ (6) is the same corpus spiral gap $(\pi - 3)^2$ [12], applied to the observed turn of the disk. Epistemic status: a dimensionless observer-invariant quantity (level L2).

## IV.5. Statement of Proposition 1

**Proposition 1 (The projective pole of rotation on the $\nu_\Phi$ spectrum).** Let an observer with a single sampling frequency $\nu_{\text{obs}}$ observe a disk rotating at angular speed $\omega$ through a rank-limited operator $\hat{O}_B$ (a single projector $P_A$). Then, under assumptions A1–A4, the apparent limits "the disk is static" ($\nu_\Phi \to 0$: strobe matching or the axisymmetric indistinguishability of the mass density) and "the disk is everywhere at once" ($\nu_\Phi \to \infty$) are two antipodal affine charts $\chi_0, \chi_\infty$ of the $\nu_\Phi$ spectrum, identified by the Möbius inversion $\iota_M$ into a single projective point $[0 : \infty] \in \mathbb{R}P^1$. The choice of chart is an artifact of the pair $(\nu_{\text{obs}}, \text{disk})$; the point $[0 : \infty]$ itself is structural and invariant.

**Assumptions. A1** — the operator $\hat{O}_B$ is rank-limited to a single projector $P_A$ (a single observation frequency); **A2** — the disk is rigid and axisymmetric in mass density (the stasis-reading degeneracy is well-defined); **A3** — the $\nu_\Phi$ spectrum admits the same continuous projective extension as Theorem 1 of the work [2] (the angular analogue of Lemma L1); **A4** — no completion of the basis (a single chart only): the level-L2/L3 invariants are excluded from the single-frequency reading by construction and re-enter in §V under basis completion.

The full derivation chain of Proposition 1 from assumptions A1–A4, via the projective-extension lemma (the angular analogue of Lemmas L1–L4 on the intrinsic rest frame of light), is given in [2]; there the structural lift "photon $\to$ disk" is recorded explicitly, and the Möbius identification (4) is shown to be the same operation, a single shared construction. Epistemic status of Proposition 1: a NEW result (T4), the angular transposition of Theorem 1 [2].

# V. THE INVARIANTS OF THE COMPLETE BASIS AND COROLLARY 1

## V.1. The reading under basis completion

Assumption A4 restricts Proposition 1 to a single chart. When the operator $\hat{O}$ is completed to the full basis (the stable fixed point $\Psi^* = \Phi(\Psi^*)$ realizing rest), the single-frequency degeneracy lifts, and the genuine invariants appear: the angular momentum $L$, the Sagnac phase [4], and the Lense–Thirring dragging of inertial frames [5]. These quantities are invariants of levels L2/L3 — observer-invariant and chart-independent. The complete basis realizes rest; the angular momentum $L$ survives the complete basis. The quaternion realization of orientation [9] sets the angular momentum through the action $R = q \cdot \Psi \cdot \bar{q}$: the quaternion orients the observer in the configuration space, and this orientation persists under basis completion.

## V.2. The three-level distinction of the status of motion

The status of the disk's motion is separated into three epistemic levels [1, 2]. Apparent stasis and apparent counter-rotation are the chart-dependent observed reading (level

L1, the sampling convention). The angular momentum $L$, the Sagnac phase, and Lense–Thirring dragging are structural invariants (levels L2/L3). Motion is chart-dependent at level L1 and invariant-real at levels L2/L3.

The self-undermining test of the thesis is essential. Were rotation a pure illusion, the angular momentum $L$, the Sagnac phase, and Lense–Thirring dragging would vanish under basis completion; they survive — hence motion is chart-dependent at level L1 and invariant-real at levels L2/L3. A ring Sagnac interferometer measures a phase difference independent of the observer's sampling frequency, fixing the L2/L3 status of rotation operationally.

## V.3. Statement of Corollary 1

**Corollary 1 (The $(\pi-3)^2$ residue of the observed turn).** Upon a complete $2\pi$ turn through the rank-limited chart $\chi_\nu$, the observed angular measure carries the spiral residue $(\pi-3)^2$ — the structural discrepancy between the continuous $2\pi$ turn and its discretely-observed reconstruction. The residue $(\pi-3)^2 \approx 0.0200$ is dimensionless, observer-invariant, and coincides with the corpus spiral gap [12].

Corollary 1 uses the quantity (6) defined in §IV and evaluates it here. The value of the residue to 50 significant digits (computed via `mpmath`, `mp.dps = 50`, with no fitting):

$$(\pi-3)^2 = 0.020048479550599188058630700199133830130683010990156. \tag{7}$$

This is the same corpus invariant as the closure ceiling $S^{\max} = 1-(\pi-3)^2$ [12]; no new fitting is introduced. $(\pi-3)^2$ is the structural discrepancy between the continuous $2\pi$ turn and its discrete reconstruction — dimensionless and observer-invariant.

Translational speed bounds lie outside the scope of the present work and are treated in other parts of the corpus.

# VI. FALSIFIERS

The claims of the work are falsifiable in three independent regimes; openness to refutation is a substantive part of the thesis.

**F-1 (numerical).** The residue $(\pi-3)^2$ (7) is reproduced at 50-digit precision through a direct recomputation; a deviation beyond machine precision refutes Corollary 1.

**F-2 (decisive).** The Sagnac phase and the angular momentum $L$ remain independent of the observer's sampling frequency. An experiment in which an invariant of the rotating disk shifts with a change of the observer's sampling frequency refutes the thesis. Operational form: a phase-difference measurement in a ring Sagnac interferometer [4] is independent of the observer's sampling frequency.

**F-3 (parsimony).** The projective reading of the pole of rotation introduces no free parameters beyond the corpus constants $(\pi-3)^2$ and $\varphi$. The introduction of an additional free parameter refutes the claimed economy.

# VII. LIMITATIONS

First, the spectral reading of the operator $\hat{H}$ (the angular Hamiltonian as the generator of rotation) is held at the status of a hypothesis and is placed outside the theorem box: the link of the spectrum of $\hat{H}$ to the projective point $[0 : \infty]$ is offered here as open to revision [HYPOTHESIS].

Second, the single-frequency idealization (assumptions A1–A4) is a limiting case. Real multi-frequency observation selects several charts simultaneously, and the projective point $[0 : \infty]$ is the structural limit of this family of charts.

Third, translational speed bounds lie outside the scope of the present work (they are treated in other parts of the corpus); no dimensional angular constant and no front speed are introduced here.

Finally, the interpretation of the residue $(\pi - 3)^2$ as a "reconstruction error of the observed turn" is a corpus-consistent reading, held at the same status of a testable prediction as the $\varphi$-embedding of the work [12]. Dimensional estimates (characteristic times, the linear sizes of the disk) are marked as phenomenological anchors and are nowhere presented as derivations from $\pi$ and $\varphi$.

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# ODTOE Corpus Navigation

Full corpus of the author's articles: odtoe.org/en/articles.

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