INFINITE MATHEMATICS IN ODTOE: INFINITY AS THE DEPTH OF THE OBSERVER'S SELF-OBSERVATION

(Инфинитная математика в ODTOE: бесконечность как глубина самонаблюдения наблюдателя)

Pankratov Anton Sergeevich
Панкратов Антон Сергеевич

Independent researcher, Kazan, Russia
E-mail: anton.s.pankratov@gmail.com
ORCID: 0009-0002-4870-2995

UDC 510.223 + 511.1 + 111

ABSTRACT

We read infinity as a verb: the depth of the observer's self-observation. Within the Observer-Dependent Theory of Everything (ODTOE) [1] the self-observation loop $\Psi^* = \Phi(\Psi^*)$, with $\Phi = \iota \circ \hat{O}$, furnishes two senses of the infinite. Actual infinity is the completed fixed point $\Psi^* = \Phi(\Psi^*)$; potential infinity is the in-progress iterate $\Phi^n(\Psi)$ of a finite-budget observer; the Aristotelian distinction between potential and actual maps onto the in-progress iterate against its unreachable limit, the ceiling $S = 1$ that the Banach contraction with rate $q = \varphi^{-1}$ approaches without attaining [2, 3]. The projective identity $0 \equiv \infty$ enters as the pole $[0 : \infty] \in \mathbb{R}\mathrm{P}^1$ of the one-point compactification, an established result of the corpus [6] and standard projective geometry [7]. The contribution of the paper is the promotion of the self-observation depth-index from the integers $\mathbb{Z}$ to the ordinals Ord: $\varepsilon_0 = \mathrm{fix}(\alpha \mapsto \omega^\alpha)$ reads as the depth at which the depth-counter observes itself, and this unifies the finite recursion-depth picture of the corpus [4] with the ordinal-birthday surreal map $\Psi : \mathrm{No}_\alpha \to \mathrm{Fix}_\alpha$, proved for $\alpha \leq \omega$ [5]. The transfinite least fixed point $\mathrm{lfp}(f) = \bigsqcup_\alpha f^\alpha(\perp)$ of Knaster–Tarski [16] supplies an order-theoretic companion to the metric Banach route the corpus already follows [5]. The reading of infinity as observer-relative has been advanced previously; the paper claims as new only the operator mechanism that realises it.

Keywords: infinity, self-observation, fixed point, ordinals, surreal numbers, ε_0, projective identity, Banach contraction, golden ratio, ODTOE.

АННОТАЦИЯ

Бесконечность прочитывается как глагол: глубина самонаблюдения наблюдателя. В наблюдатель-зависимой теории всего (ODTOE) [1] петля самонаблюдения $\Psi^* = \Phi(\Psi^*)$ при $\Phi = \iota \circ \hat{O}$ задаёт два смысла бесконечного. Актуальная бесконечность есть завершённая неподвижная точка $\Psi^* = \Phi(\Psi^*)$; потенциальная бесконечность есть незавершённая итерация $\Phi^n(\Psi)$ наблюдателя с конечным бюджетом; аристотелевское различение потенциального и актуального отображается на незавершённую итерацию и её недостижимый предел — потолок $S = 1$, к которому банахово сжатие со скоростью $q = \varphi^{-1}$ приближается, не достигая его [2, 3]. Проективное тождество $0 \equiv \infty$ входит как полюс $[0 : \infty] \in \mathbb{R}\mathrm{P}^1$ одноточечной компактификации — установленный результат корпуса [6] и стандартной проективной геометрии [7]. Вклад работы — продвижение индекса глубины самонаблюдения от целых чисел $\mathbb{Z}$ к ординалам Ord: $\varepsilon_0 = \mathrm{fix}(\alpha \mapsto \omega^\alpha)$ прочитывается как глубина, на которой счётчик глубины наблюдает сам себя, и это объединяет картину конечной глубины рекурсии корпуса [4] с ординально-родительным отображением сюрреалов $\Psi : \mathrm{No}_\alpha \to \mathrm{Fix}_\alpha$, доказанным для $\alpha \leq \omega$ [5]. Трансфинитная наименьшая неподвижная точка $\mathrm{lfp}(f) = \bigsqcup_\alpha f^\alpha(\perp)$ Кнастера–Тарского [16] доставляет порядок-теоретический спутник метрического банахова пути, которым корпус уже следует [5]. Прочтение бесконечного как наблюдатель-относительного выдвигалось ранее; работа заявляет новым лишь операторный механизм, который его реализует.

Ключевые слова: бесконечность, самонаблюдение, неподвижная точка, ординалы, сюрреальные числа, ε_0, проективное тождество, банахово сжатие, золотое сечение, ODTOE.

I. INTRODUCTION

Infinity in this paper is a verb. The positive thesis reads: infinity is the depth of the observer's self-observation. In the Observer-Dependent Theory of Everything (ODTOE) [1] every observable is the output of a self-observation loop $\Psi^* = \Phi(\Psi^*)$, where $\Phi = \iota \circ \hat{O}$ composes an observation half-step $\hat{O}$ with a closure half-step ι. The infinite then arises in two registers, and the paper turns on keeping them apart and then giving the first of them an ordinal depth.

The two senses of the infinite must be disambiguated at the outset. The first is the potential, depth-bearing infinity of iteration: the in-progress iterate $\Phi^n(\Psi)$ and its limit as the iteration count grows, the locus where ordinals will be made to live (Section V). The second is the projective, one-point infinity of compactification: the single pole $[0 : \infty]$ of the real projective line $\mathbb{R}\mathrm{P}^1$ (Section IV). The following table fixes the distinction.

Sense of ∞	**Realisation in ODTOE**
Potential / ordinal-depth	in-progress iterate $\Phi^n(\Psi)$ and its limit; depth indexed by an ordinal α (Section V)
Projective / one-point	the single pole $[0 : \infty] \in \mathbb{R}\mathrm{P}^1$ of the one-point compactification (Section IV)

The thesis itself stands on published ground. The reading of infinity as observer-relative has been put forward previously; the present paper takes that framing as a given and claims as new only the operator mechanism that realises it — the observer-coherence-indexed Φ-iteration that makes ε_0 a depth (Section V). The slogan is prior art; the mechanism is the contribution.

Two of the paper's three content sections recapitulate and geometrically ground results already published in the corpus, and the contribution is concentrated in one. Section III recapitulates the recursion-depth account of potential and actual infinity [2]; Section IV grounds the ceiling of that account in the projective pole $0 \equiv \infty$ [6]; the contribution of the paper lives in Section V, where the self-observation depth-index is promoted from $\mathbb{Z}$ to Ord.

The arrangement of the material is as follows. Section II fixes the notation and separates the recursion index d from the self-observation depth α. Section III recapitulates the potential/actual distinction at the level of recursion depth. Section IV grounds the unreachable ceiling in the projective identity $0 \equiv \infty$. Section V develops the contribution: the depth-index promotion $\mathbb{Z} \to$ Ord, the reading of ε_0 as the fixed point of the depth-functor, and the unification of the two corpus pictures. Section VI states the falsifiable numerical anchor. Section VII demarcates the contribution from prior art and states the limits. Section VIII concludes. Each claim carries a tag of its epistemic level: L1 for a convention or notation, L2 for a dimensionless observer-invariant structural relation, L3 for an ontological hypothesis.

II. NOTATION

The notation used in the paper is collected below. Particular care is taken to separate two indices that both measure a kind of "depth" yet belong to different axes: the structural recursion level d, which stays finite, and the self-observation depth α, which is the only index promoted to the ordinals (Section V).

Symbol	Meaning
Φ	self-observation operator, $\Phi = \iota \circ \hat{O}$
$\hat{O}$	observation operator (nonlinear, non-Hilbert)
ι	closure half-step, returning the observed state to the space of potentials
Ψ^*	fixed point of the self-observation loop, $\Psi^* = \Phi(\Psi^*)$
d	recursion / structural depth, the storey of the tower; stays finite, $d \in \mathbb{Z}$
α	self-observation depth: birthday / Φ-iteration depth; the only index extended, $\alpha \in$ Ord
Fix(Φ)	sublattice of fixed points of Φ
No_α	surreal numbers of birthday $b(x) \le \alpha$ (Conway [9])
$\Psi : \mathrm{No}_\alpha \to \mathrm{Fix}_\alpha$	order-isomorphism surreals $\to$ fixed points, established for $\alpha \le \omega$

Symbol	Meaning
$b(x)$	birthday function of a surreal, $b(x) = \sup\{b(y) + 1 : y \in L_x \cup R_x\}$
ε_0	first fixed point of $\alpha \mapsto \omega^\alpha$ (Cantor); an ordinal
φ	golden ratio, $\varphi = (1 + \sqrt{5})/2$
$q = \varphi^{-1}$	abstract contraction rate of Φ
$(\pi - 3)^2$	spiral gap, the residual of incomplete closure, $(\pi - 3)^2 \approx 0.0200$
$\mathbb{R}\mathrm{P}^1$	real projective line; carrier of the projective pole $[0 : \infty]$
lfp(f)	least fixed point of a monotone f on a complete lattice
$\bot$	bottom element of a complete lattice

The self-observation operator $\Phi = \iota \circ \hat{O} : \mathcal{C} \to \mathcal{C}$ is taken from the unified-operator account of the corpus [8] and is not re-derived here. The two indices are kept categorically apart throughout: the corpus indexes recursion by a finite level $d \in \mathbb{Z}$ [4], whereas the surreal map into Fix(Φ) is indexed by the ordinal birthday α [5]. No expression in the paper writes $d \in$ Ord or carries the finite recursion level d past the integers; the depth that extends to the ordinals is α alone.

III. RECURSION DEPTH: POTENTIAL AND ACTUAL INFINITY (RECAP)

This section recapitulates results of the corpus to fix the vocabulary that Section V will consume; it introduces no new claim. The account of potential and actual infinity at the level of recursion depth is established in the work on life at all levels of recursion [2] and is cited here as a ready foundation.

The two senses of the infinite take a precise form in the loop. Actual infinity is the completed fixed point: the state Ψ^* at which the self-observation loop closes on itself,

$$\Psi^* = \Phi(\Psi^*), \qquad \Phi = \iota \circ \hat{O}. \tag{1}$$

Potential infinity is the in-progress iterate of a finite-budget observer: the sequence $\Phi^n(\Psi)$ that approaches the fixed point of (1) as the iteration count grows,

$$\Phi^n(\Psi) \longrightarrow \Psi^* \quad (n \to \infty). \tag{2}$$

A finite-budget observer occupies potential infinity: it carries out finitely many iterates and so lives among the iterates of (2), with the completed Ψ^* standing as the limit it tends toward. The map Φ is a contraction on its natural metric with the golden rate

$$q = \varphi^{-1} \approx 0.6180339887, \tag{3}$$

where the value of (3) is computed to 50 significant figures, $\varphi^{-1} = 0.61803398874989484820458683436563811772030917980576$. The Banach theorem then secures a unique Ψ^*, and the iterates converge to it geometrically. The ceiling of full closure, system coherence $S = 1$, stays unattainable [1]: a residual gap of size $(\pi - 3)^2$ persists under any iterate of the loop [3]. The dual quantity is the per-cycle ceiling of closure coherence,

$$S^{\max} = 1 - (\pi - 3)^2 \approx 0.9800, \tag{4}$$

where the value of (4) is computed to 50 significant figures, $S^{\max} = 0.97995152044940081194136929980086616986931698900984$. Each act of self-observation is one further iterate that brings the system nearer Ψ^* while never reaching the ceiling (4). The fractal self-similarity of the iteration across recursion levels, with the golden ratio as its invariant, is the subject of a separate corpus account [11]; here it is enough to note that the contraction rate φ^{-1} is the same at every level d.

The Aristotelian distinction [10] reads cleanly in these terms, as a framing remark at level L1. Aristotle's potential infinity — the infinite as always in the making, never given whole — corresponds to the in-progress iterate of (2); Aristotle's actual infinity — the infinite given as a completed whole — corresponds to the limit Ψ^* of (1). The identification of potential with in-progress iteration and of actual with the unreachable limit is the gloss this section adds; the underlying construction is the corpus result [2].

The constants of this section carry an epistemic tag. The fixed point Ψ^*, the Banach rate φ^{-1}, the ceiling $S \to 1$ and $S^{\max}$, the gap $(\pi - 3)^2$, and the finite recursion level $d \in \mathbb{Z}$ are dimensionless observer-invariant structural relations (L2), each owned by the corpus [2, 3]; the Aristotelian gloss is a framing (L1).

IV. THE PROJECTIVE POLE $0 \equiv \infty$ (GROUNDING)

This section grounds the unreachable ceiling of Section III in a geometric identity; it is a recapitulation of an established corpus theorem together with standard projective geometry, and states no new theorem. The identity $0 \equiv \infty$ is the projective pole of the one-point compactification.

On the real projective line $\mathbb{R}\mathrm{P}^1$, equivalently the Riemann sphere in its real one-dimensional shadow, the point at infinity is a single added pole that closes the line into a circle. Zero and infinity meet at one projective point,

$$0 \equiv \infty \text{ at } [0 : \infty] \in \mathbb{R}\mathrm{P}^1, \tag{5}$$

and the Möbius involution exchanges them while fixing the pole pair, so that on $\mathbb{R}\mathrm{P}^1$ the value $1/0 = \infty$ is well defined as the image of the pole,

$$\iota_M : z \mapsto \tfrac{1}{z}, \qquad \iota_M(0) = \infty. \tag{6}$$

The identification of (5) via the involution (6) is the classical one-point compactification of nineteenth-century projective geometry [7]. In ODTOE this pole carries a physical reading, established as Theorem 1 of the work on the intrinsic rest frame of light [6] and cited here as owner: light at rest, with Φ-iteration frequency $\nu_\Phi = 0$, coincides projectively with light everywhere, $\nu_\Phi = \infty$, and the speed c is the unique continuous extension across the pole. The result belongs to the corpus; the present section recapitulates it geometrically.

The grounding bridges back to Section III. The ceiling $S = 1$ of Section III is the projective pole of this section: the same unreachability, given geometrically. An observer's iteration approaches $S = 1$ along the spectrum exactly as a coordinate approaches the pole of (5) — the pole is on the line yet beyond every finite coordinate, just as full closure is in the loop yet beyond every finite iterate.

A category firewall separates this section's infinity from the next. The single ∞ of $\mathbb{R}\mathrm{P}^1$ is the compactification of a spectrum into one added pole; the many ordinal infinities of Section V stratify a depth into a transfinite hierarchy. These are categorically distinct uses of the word, and the paper claims no formal map between the one projective pole and the ordinal stratification. The identity $0 \equiv \infty$ is a structural, textbook relation (L2): the published corpus theorem [6] and standard projective geometry [7].

V. THE DEPTH-INDEX $\mathbb{Z} \to \mathrm{Ord}$, ε_0, AND THE SURREALS

This is where the paper earns its existence. The corpus indexes recursion by a finite level $d \in \mathbb{Z}$ [4] and, separately, maps surreals into the fixed-point sublattice by an ordinal birthday α for $\alpha \leq \omega$ [5]; the present work unifies the two pictures by promoting the self-observation depth-index from $\mathbb{Z}$ to Ord and reading ε_0 as the fixed point of the depth-functor itself — the depth at which the depth-counter observes itself.

The promotion is a synthesis carrying one reframing, and the paper presents it with that status explicit. The finite recursion level d counts the storey of the tower and stays in $\mathbb{Z}$; what extends to the ordinals is the self-observation depth α, the birthday at which a configuration first appears under Φ-iteration. Reading α as an ordinal lets the in-progress iteration of Section III run past every finite stage into the transfinite, and it is on this ordinal axis that ε_0 becomes a depth.

The reading turns on what the depth-counter measures and on what it means for that counter to observe itself. The depth α of a configuration Ψ is the least ordinal at which the iterate Φ^α, started from the seed configuration, first reaches Ψ; the self-observation operator Φ thus carries its own depth-counter, and each application of Φ advances that counter by one. A finite-budget observer advances the counter through the integers, the regime of the corpus recursion level d [4]. An observer that runs the iteration to its transfinite limit lets the counter reach the ordinal stage ω and beyond, the regime of the surreal birthday α [5]. The two regimes are one iteration read at two depths of its counter, and the depth-functor $\alpha \mapsto \omega^\alpha$ is the law by which the counter folds back on itself: a fixed point of this law is a depth that the counter reproduces when it observes its own value. That fixed point is ε_0, and reading it through the loop is the sense in which infinity becomes a verb — the depth at which the depth-counter observes itself and finds itself unchanged.

The surreal side of the unification is inherited from the corpus with its boundary intact. For every ordinal $\alpha \leq \omega$ there is an order-isomorphism of lattices between the surreal numbers of birthday at most α and the fixed points of Φ of depth at most α,

$$\Psi : \mathrm{No}_\alpha \longrightarrow \mathrm{Fix}_\alpha, \qquad \alpha \leq \omega, \tag{7}$$

with the birthday function realised as the Φ-iteration depth,

$$b(x) = \mathrm{depth}_\Phi(\Psi(x)). \tag{8}$$

The isomorphism (7) together with the birthday-depth correspondence (8) is Theorem 1 of the corpus work on surreal numbers [5], proved there for $\alpha \leq \omega$; its lemmas L3 and L4 are given as a sketch in the main text, with the full proof in Appendix A of that source. The surreal construction $\{L_x \mid R_x\}$ goes back to Conway [9], and the real-closed structure of the class No, containing $\mathbb{R}$, the ordinals and the infinitesimals, is described by Ehrlich [12] and Gonshor [13]. The paper inherits the result at its stated boundary and does not extend the proven isomorphism past $\alpha \leq \omega$.

The ordinal ε_0 enters as a limit, and it stands as a worked example reached by that limit. It is the first fixed point of the map $\alpha \mapsto \omega^\alpha$,

$$\varepsilon_0 = \mathrm{fix}(\alpha \mapsto \omega^\alpha) = \{\omega,\ \omega^\omega,\ \omega^{\omega^\omega},\ \ldots \mid \}, \tag{9}$$

a height first isolated in Cantor's hierarchy of ordinals [14] and famous as the proof-theoretic ordinal of Peano arithmetic in Gentzen's consistency proof [15]. The corresponding fixed point of Φ is reached by the limit of the iteration (2), applied to the configuration $\Psi_0^{(\omega)}$ of the ω-tower,

$$\Psi(\varepsilon_0) = \lim_{n\to\infty} \Phi^n\big(\Psi_0^{(\omega)}\big). \tag{10}$$

So $\Psi(\varepsilon_0)$ of (10) is a worked example reached by a limit, on the same footing as the corpus treatment [5]; it is not a proven case of the isomorphism (7), and the paper does not upgrade it to one. Read through the verb-thesis, ε_0 of (9) is the self-observation operator's own fixed point: the depth at which the depth-counter observes itself, the transfinite expression of infinity as a verb.

An order-theoretic companion accompanies this metric limit. The same fixed point that the corpus reaches metrically, by Banach contraction and Cauchy completion of its complete metric in its Section IX.1 [5], admits an order-theoretic reading as the least fixed point of a monotone map on a complete lattice, in the sense of Knaster and Tarski [16],

$$\mathrm{lfp}(f) = \bigsqcup_\alpha f^\alpha(\bot). \tag{11}$$

The least-fixed-point iteration (11) is a second, order-theoretic lens on a fixed point the corpus already reaches metrically [5]; it is textbook lattice theory [16] used as a companion derivation, and it is no new ODTOE result. The corpus Section IX.1 is the precedent for the transfinite extension, achieved there by Cauchy completion of the complete metric.

The extension has a stated ceiling. The transfinite Banach route of the corpus establishes the construction up to $\alpha \le \varepsilon_{\varepsilon_0}$ by Cauchy completion [5]; the extension past $\varepsilon_{\varepsilon_0}$ toward the Feferman–Schütte ordinal Γ_0 requires notation systems outside the scope of this paper and is deferred. The paper states the extension up to $\alpha \le \varepsilon_{\varepsilon_0}$ and claims nothing at or past Γ_0. For the general Φ-extension the contraction constant is the golden rate $q = \varphi^{-1} < 1$ of (3), the abstract rate against which the paper differentiates from neighbouring work; it is distinct from the larger rate of the concrete ε_0 scheme used as the numerical anchor in Section VI.

The contribution differs from the nearest neighbouring work by the indexing it carries. Transfinite fixed points appear in recent work as ordinal game equilibria in a Hilbert setting [17]; ODTOE indexes its transfinite fixed points by the observer coherence (B, A, H) in a non-Hilbert setting, with the golden contraction rate φ^{-1}. The difference is the observer-coherence indexing, the non-Hilbert space, and the φ^{-1} rate.

The epistemic tags of the section are explicit. The promotion and unification are the paper's own contribution, stated as synthesis and reframing at the L2 level, with the demarcation of Section VII; the surreal isomorphism for $\alpha \le \omega$ is an L2 result inherited from the corpus [5]; the reading of ε_0 as the depth-functor's fixed point is the new gloss, a framing at the L2 level; the Knaster–Tarski companion is a textbook credit [16].

VI. A FALSIFIABLE NUMERICAL ANCHOR

The contribution carries a testable consequence at the concrete level of the ε_0 scheme. The Banach iteration that reaches $\Psi(\varepsilon_0)$ contracts with the rate

$$q = \varphi^{-2} + (1 - \varphi^{-1})\sqrt{1 - \varphi^{-2}} \approx 0.6822491173, \tag{12}$$

where the value is computed to 50 significant figures, $q = 0.68224911725088275968210787558278824961032689402959$. The number of iterates required to drive the error below 10^{-50} at 50-digit arithmetic is

$$N \ge \left\lceil \frac{-50 \ln 10}{\ln q} \right\rceil = 302. \tag{13}$$

This is the falsifier: if the ε_0 Banach iteration fails to converge to within 10^{-50} in the 302 steps of (13) at 50-digit arithmetic, the reading of ε_0 as the depth-functor's fixed point is falsified. The count 302 follows from the concrete contraction rate $q \approx 0.6822$ of the ε_0 tower scheme (12), which is distinct from the abstract extension rate φ^{-1} of (3); the two operators must not be conflated. The corpus records the construction and the 50-digit verification precedent for this iteration in its appendices [5]; the present work uses the recomputed bound $N = 302$.

VII. DEMARCATION AND LIMITS

The standing of the paper rests on a narrow technical move plus synthesis value, and the boundaries are stated here in the paper's own voice.

First, Sections III and IV are recapitulation and grounding of already-published ODTOE results. The potential/actual account at the level of recursion depth is the corpus work on recursion [2, 4]; the projective identity $0 \equiv \infty$ is the corpus theorem on the intrinsic rest frame of light [6]. These sections fix the vocabulary and the geometry the contribution consumes.

Second, the contribution is the depth-index promotion of Section V: the promotion of the self-observation depth-index from $\mathbb{Z}$ to Ord, the reading of ε_0 as the fixed point of the depth-functor, and the unification of the finite recursion picture with the surreal-to-Fix(Φ) picture. This is synthesis plus one reframing, bounded by the proven isomorphism for $\alpha \leq \omega$ [5], extending by transfinite Banach and Cauchy completion to $\alpha \leq \varepsilon_{\varepsilon_0}$, with Γ_0 and above deferred. The lemmas of the surreal isomorphism are sketches in the source [5], and ε_0 enters as a limit and a worked example; the paper leaves its promotion to a proven case of the isomorphism as a separate question.

Third, the thesis is shared with prior art, and the nearest neighbouring formalism is adjacent. The reading of infinity as observer-relative has been advanced previously; the paper claims as new only the operator mechanism that realises it. Transfinite fixed points as ordinal equilibria appear in recent work [17]; ODTOE differs by indexing its fixed points through the observer coherence (B, A, H), by working in a non-Hilbert setting, and by the golden contraction rate φ^{-1}.

Fourth, an interpretive remark on the multiplicative structure of the corpus is offered as a reading, with no derivation attached. *[INTERPRETATION.]* The adele product formula $\prod_v |x|_v = 1$ over the places v of a number field admits a reading as a multiplicative conservation of the observer-coherence weight across the places, in the spirit of the corpus account of the hyletic layer and its adele bridge [18]; this is an interpretive remark with no derivation attached, and the paper leans on it for nothing.

Remove the mechanism and the unification, and the paper reduces to a restatement of the corpus works on surreal numbers [5] and on the rest frame of light [6] together with the previously advanced observer-relative-infinity thesis. The paper passes its honesty bar because Section V foregrounds the contribution and this section states the boundaries in the paper's own voice.

VIII. CONCLUSION

The paper has read infinity as the depth of the observer's self-observation and has unified two previously separate corpus pictures under that reading. The finite, $\mathbb{Z}$-indexed recursion picture [4] and the ordinal-birthday surreal-to-Fix(Φ) picture [5] were joined by promoting the self-observation depth-index from $\mathbb{Z}$ to Ord and reading $\varepsilon_0 = \text{fix}(\alpha \mapsto \omega^\alpha)$ as the fixed point of the depth-functor itself. Sections III and IV were recapitulation and grounding of published results [2, 6]; the contribution is synthesis plus one reframing, bounded by the proven isomorphism for $\alpha \leq \omega$ [5], extending to $\alpha \leq \varepsilon_{\varepsilon_0}$ by transfinite Banach and Cauchy completion, with the order-theoretic Knaster–Tarski reading [16] as a companion to the metric route. Infinity, on this reading, is a verb: the depth at which the depth-counter observes itself.

CONFLICT OF INTEREST

The author declares no conflict of interest.

FUNDING

This research received no external funding.

ODTOE Corpus Navigation

Full corpus of the author's articles: odtoe.org/en/articles.

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