Coherence of the Human–AI Learning Dyad in Adaptive Learning: the Zone of Proximal Development as an Interior Optimum of the Control Parameter

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Abstract

This paper treats the learner and AI-tutor dyad as a single coherent learning system. The learner's state is denoted Ψ, the target competence Ψ^*, and learning is read as convergence to a self-consistent fixed point. The coherence of the dyad is given by the multiplicative anchor $B = F \cdot E \cdot (1-\sigma) \cdot \Lambda$ with a weak-link property: zeroing any factor zeroes B. The difficulty of presented material has an interior optimum ρ^* lying in the intersection of the zone of proximal development and the flow state. Progress is measured through the reduction of the mastery gap. A coherent and convincing appearance of mastery forms the ideal error δ_{ideal}: high coherence raises the learner's confidence, while factual mastery remains a distinct quantity. The self-regulated learning loop (forethought, performance, reflection) is closed with the AI as a co-regulator. Collective reliance S on the tutor has an interior optimum of reliability at which the learner remains the driver. Six operationalizable proxy metrics, a table of spine-versus-periphery correspondences, and a falsifiable program of eight predictions and one hypothesis are proposed. All cross-domain correspondences are held as structural analogies at the level of control-parameter topology. All of mathematics, physics, and the phenomenology of consciousness are projections of a single primary act of distinction.

Keywords: adaptive learning, dyad coherence, human–AI, self-observation fixed point, interior optimum, zone of proximal development, flow, mastery gap, ideal error, self-regulated learning, ODTOE.

Аннотация. Работа рассматривает диаду «ученик и ИИ-наставник» как единую когерентную обучающую систему. Состояние ученика обозначается Ψ, целевая компетентность — Ψ^*, а само обучение трактуется как сходимость к самосогласованной неподвижной точке. Когерентность диады задаётся мультипликативным якорем $B = F \cdot E \cdot (1-\sigma) \cdot \Lambda$ со свойством слабого звена: обнуление любого множителя обнуляет B. Сложность подачи материала имеет внутренний оптимум ρ^*, лежащий в полосе пересечения зоны ближайшего развития и состояния потока. Прогресс измеряется через сокращение разрыва владения, а связная и убедительная видимость освоения образует идеальную ошибку δ_{ideal}: высокая когерентность повышает уверенность ученика, тогда как фактическое владение остаётся иной величиной. Петля саморегулируемого обучения (предусмотрение, исполнение, рефлексия) замыкается с ИИ как со-регулятором. Коллективная опора S на наставника имеет внутренний оптимум надёжности, при котором ведущим остаётся ученик. Предложены шесть операционализируемых прокси-метрик, таблица соответствий хребта и периферии и фальсифицируемая программа из восьми предсказаний и одной гипотезы. Все междоменные соответствия удерживаются как структурные аналогии на уровне топологии управляющего параметра. Вся математика, физика и феноменология сознания суть проекции единого первичного акта различения.

Ключевые слова: адаптивное обучение, когерентность диады, человек–ИИ, неподвижная точка самонаблюдения, внутренний оптимум, зона ближайшего развития, поток, разрыв владения, идеальная ошибка, саморегулируемое обучение, ODTOE.

1 Introduction: the human–AI dyad as a single learning system

Adaptive learning with an AI tutor is considered here as the behaviour of a single coherent system formed by the learner and the tutor. The approach developed below is abbreviated ODTOE (Observer-Dependent Theory of Everything); within this paper it is a metatheoretical framework that parametrizes the space of learning descriptions through the coherence of the learner-as-observer. The central thesis is stated affirmatively: the learner and the AI tutor form a coherent dyad whose state converges to a self-consistent fixed point of competence with an interior optimum of difficulty. The classical zone of proximal development [1] provides the pedagogical anchor: development takes place in the band between what the learner can already do alone and what is reachable with support. The flow state [2] provides the motivational anchor of the same band: engagement is sustained where task difficulty is matched to the current skill.

Every major claim in this work carries an epistemic level. **L2-INVARIANT** marks a structural result that transfers across domains. **PREDICTION** marks an empirically testable consequence of the model. **HYPOTHESIS** marks a statement that is open within the corpus or imported from an adjacent field. The organizing object of the whole work is the coherent fixed point of the learner's self-observation,

$$\Psi \in \mathcal{H}, \qquad \Psi^* \in \mathcal{H}, \tag{1}$$

where Ψ denotes the learner's current knowledge state in the space $\mathcal{H}$, and Ψ^* sets the target competence. The operator apparatus of learner coherence and its metrology are borrowed from the engineering reading of ODTOE [3].

Analogy, not identity. The correspondence between the pedagogical band of development, the motivational flow, and the operator topology of the control parameter holds at the level of structural analogy: common across them is the shape of a landscape with an interior peak, while the psychometric and operator quantities differ. The fixed-point operator Φ and the numerical value of the optimum ρ^* carry the status of **HYPOTHESIS**; no identification of quantities is made here.

2 Learning as convergence to a competence fixed point

The learner's target competence is set by the fixed point of self-observation,

$$\Psi^* = \Phi(\Psi^*) = \iota(\hat{O}_\Psi(\Psi)), \tag{2}$$

where the learner's state and the target competence are taken in the space (1), the self-observation operator $\hat{O}_\Psi$ describes how the learner assesses their own knowledge, and the integration operator ι ties the learner's internal state to the external field of learning material and tutor feedback. In this reading, learning is convergence to a self-consistent state: the learner can do what they can do and consistently knows that they can do it. The existence and uniqueness of the fixed point of the operator Φ remain an open problem of the corpus and are held at the level of **HYPOTHESIS** [3].

The coherence of the learning dyad is set by a multiplicative anchor over four factors,

$$B = F \cdot E \cdot (1 - \sigma) \cdot \Lambda, \tag{3}$$

where F denotes focus (keeping the learner on task), E encodes the motivational charge, $(1 - \sigma)$ sets the confidence factor (low internal mismatch), and Λ accounts for the integration of new knowledge with what is already mastered. The multiplicative form yields the weak-link property: a growing mismatch $\sigma \to 1$ drives coherence to zero,

$$\sigma \to 1 \;\Rightarrow\; B \to 0, \tag{4}$$

regardless of F, E, and Λ. The weak link dictates a diagnostic priority: restoring coherence begins with the smallest factor (**L2-INVARIANT**) [3]. The anchor of one-to-one tutoring sets the upper bound of the effect: individual work with a tutor raises a group's average outcome by about two sigma relative to frontal instruction [4]; the AI tutor carries this one-to-one regime to scale.

Analogy, not identity. The factors F, E, $(1-\sigma)$, Λ are operationalized as proxies (Section 7) and carry the status of **HYPOTHESIS** at the level of a measurable quantity. The transfer of the multiplicative form from the engineering reading of coherence into the pedagogical domain is a structural analogy of control-parameter topology, while the psychometric and operator quantities differ.

3 The difficulty control parameter and the interior optimum ρ^*

A single control parameter ρ is introduced as the normalized difficulty of presented material relative to the learner's current skill. The topology of the action of ρ is arranged as follows. As $\rho \to 0$ the material is too easy: the learner is bored, motivation drops, progress slows. As $\rho \to 1$ the material is too hard: the learner loses footing, anxiety grows, the coherence of the dyad collapses. Between these edges lies an interior optimum ρ^*,

$$\left.\frac{\partial\,\text{Quality}}{\partial\rho}\right|_{\rho=\rho^*} = 0, \qquad 0 < \rho^* < 1, \tag{5}$$

at which the quality of learning reaches its maximum. The quality profile as a function of difficulty is single-peaked,

$$\text{Quality}(\rho) \text{ single-peaked at } \rho^*. \tag{6}$$

The optimum ρ^* coincides with the intersection band of the zone of proximal development [1] and the flow state [2]: the task is set above the learner's current independent level and remains reachable with tutor support, sustaining engagement. The role of the AI tutor is to keep the presentation continuously in the neighbourhood of ρ^* by adapting the difficulty of the next step, consistent with the single-peaked profile (6). The existence of the interior optimum is fixed as a structural result (**L2-INVARIANT**); the concrete numerical value of ρ^* depends on the learner and the subject and is held at the level of **HYPOTHESIS**.

Analogy, not identity. The correspondence of the band ZPD$\cap$Flow and the operator optimum ρ^* is a structural analogy at the level of control-parameter topology: common across them remains the shape of a landscape with an interior peak, while the psychometric scale of difficulty and the operator parameter ρ are distinct quantities; no identification of quantities is made here.

4 The signature of the ideal error: the mastery gap and the appearance of mastery

The learner's progress is measured through the reduction of the mastery gap between the target competence and the factual state,

$$\text{gap} = \|\Psi^* - \Psi\|, \tag{7}$$

and learning advances to the extent that gap shrinks step by step. The central claim of the section: high coherence of the presentation raises the learner's subjective confidence, while factual mastery remains a distinct quantity. The interval between a coherent appearance of mastery and actual mastery forms the ideal error,

$$\delta_{\text{ideal}} = \Psi^*_{\text{coherent}} - \Psi_{\text{factual}}, \tag{8}$$

where Ψ^*_{coherent} denotes a coherent and convincing «looks-mastered», and Ψ_{factual} sets actual mastery. A growth of coherence raises confidence, with mastery determined by a separate quantity,

$$\text{Coherence } \uparrow \not\Rightarrow \text{ Mastery } \uparrow. \tag{9}$$

The sources of the ideal error (8) in the learning dyad are manifold. A smooth and confident tutor explanation creates in the learner a sense of understanding that runs ahead of the ability to reproduce the material independently, in keeping with the divergence of coherence and mastery (9). Anxiety and stereotype threat lower the observed result while competence is preserved, distorting the estimate of Ψ [5]; this same channel enters the confidence factor $(1 - \sigma)$ of the anchor (3). The protection against the ideal error consists in measuring progress through the reduction of gap (7) on test tasks; the learner's subjective confidence remains a separate signal. The identification of the maximum appearance of mastery with the signature of the error is carried as a **PREDICTION** (see P4, P6 in Section 9).

Analogy, not identity. The quantities Ψ^*_{coherent} and Ψ_{factual} are operationalized as proxies of the mastery gap and carry the status of **HYPOTHESIS**. The ideal error is transferred from the engineering reading of coherent collapse into the pedagogical domain as a structural analogy, while the quantities differ.

5 The RT-2 loop of self-regulated learning with the AI co-regulator

Self-regulated learning unfolds cyclically through three phases: forethought (goal-setting and planning), performance (working on the task with self-observation), and reflection (self-evaluation and attribution of the result) [6]. This loop is read as a two-stroke circuit updating the learner's state,

$$\Psi_{t+1} = \text{Reflect}(\text{Perform}(\text{Forethink}(\Psi_t))), \tag{10}$$

where each turn reduces the mastery gap (7). The AI tutor enters the loop as a co-regulator: in the forethought phase it helps select a reachable goal in the neighbourhood of ρ^*, in the performance phase it supplies feedback and holds the focus F, in the reflection phase it supports correct attribution of the result and thereby reduces the mismatch σ.

Co-regulation transfers the regulatory function to the learner as their autonomy grows: the share of external control declines, the share of self-regulation rises. Keeping the loop active is fixed as a structural result (**L2-INVARIANT**): without the reflection phase the circuit breaks, and the state update (10) loses self-consistency [6]. From this follows the prediction that dyads with a closed reflection loop outperform dyads without one (**PREDICTION**, see P5).

Analogy, not identity. The correspondence of the three self-regulation phases and the two-stroke operator (10) is a structural analogy of update-circuit topology; the phase psychological constructs and the operator steps are distinct quantities, and no identification of quantities is made here (**HYPOTHESIS**).

6 Collective reliance and the interior optimum of reliability S^*

The learner's reliance on the AI tutor is described by a collective parameter $S \in [0,1]$ that sets the share of decisions delegated to the tutor. The topology of S repeats the topology of the control parameter. As $S \to 0$ the learner ignores available support and loses the advantage of the dyad. As $S \to 1$ the learner hands the whole work to the tutor, stops exercising the skill, and their own state Ψ stops growing. Between the edges lies an interior optimum of reliability,

$$\frac{\partial\,\text{Learning}}{\partial S}\bigg|_{S=S^*} = 0, \qquad 0 < S^* < 1, \tag{11}$$

at which the learner remains the driver of the dyad while the tutor amplifies their work. The dynamics of trust and reliance of a human on an automated aide obey the same regimes of distrust and over-trust as the interaction of a human with automation [7]: both the underuse and the abuse of support lower the joint result.

The existence of the interior optimum of reliability is fixed as a structural result (**L2-INVARIANT**); the concrete value of S^* depends on the learner, the task, and the maturity of the skill and is held at the level of **HYPOTHESIS** [7]. From this follows the prediction of a single-peaked dependence of learning on the share of reliance (**PREDICTION**, see P7): both too low and too high a reliance lower the gain in mastery.

Analogy, not identity. The reliance parameter S and the operator share of delegated control are structurally analogous quantities at the level of optimum topology; their psychometric and operator content differs, and no identification of quantities is made here.

7 Proxy metrics and the operationalization of the invariant

The applied unfolding of the model keeps the same level discipline as the theoretical part: all metrics are operationalizable proxies at the **HYPOTHESIS** level, and no correspondence here is declared an established result. Six proxy metrics form the minimal measurement circuit of the dyad. The focus factor F is read through the share of time on task. The charge factor E is read through engagement and the persistence of attempts. The confidence factor $(1-\sigma)$ is read through the calibration of self-assessment against the test result. The integration factor Λ is read through skill transfer to new tasks. The difficulty ρ is read through the error rate at the current level. The mastery gap gap is read through the result of a delayed test without hints. The summary is given in Table 1.

Table 1: Six proxy metrics of the learning dyad (operationalizable proxies; the status of each metric is HYPOTHESIS).

Metric	Operationalization (proxy)	What it tracks	Status
Focus F	Share of time on task per session	Holding of attention	HYPOTHESIS
Charge E	Engagement and persistence of attempts	Motivational factor	HYPOTHESIS
Confidence $(1-\sigma)$	Calibration of self-assessment against the test	Internal mismatch	HYPOTHESIS
Integration Λ	Skill transfer to new tasks	Coherence of knowledge	HYPOTHESIS
Difficulty ρ	Error rate at the current level	Proximity to ρ^*	HYPOTHESIS
Gap gap	Delayed test without hints	Reduction of $\|\Psi^* - \Psi\|$	HYPOTHESIS

The metric circuit ties measurement to control: the error rate and the calibration of self-assessment serve as proxies of proximity to ρ^*, while the delayed test without hints serves as a proxy of the reduction of the mastery gap (7). The principle of such reading (observable proxies of the factors and of the difficulty as control signals of the dyad) is carried with the status of **L2-INVARIANT**, while the concrete numerical estimation procedures are open and fixed at the level of **HYPOTHESIS** [3].

8 Spine and periphery: correspondences and boundaries

The model separates the spine (approaches that map directly onto the control parameter and its topology) from the periphery (approaches valuable for practice but lying outside the formal spine of the invariant). Table 2 collects the spine correspondences, Table 3 the peripheral ones.

Table 2: Spine correspondences: mapping of base approaches onto the control-parameter topology of ODTOE.

Approach	Anchor	ODTOE mapping
Flow, ZPD	Task difficulty	Interior optimum ρ^*
Self-regulated learning	Reflection cycle	RT-2 loop (10)
Self-determination	Basic needs	Form of the anchor B (3)
BKT, DKT	Skill mastery	State Ψ and gap gap
Human reliance on AI	Trust in automation	Reliance parameter S (11)
Removing the evaluative frame	Stereotype threat	Ideal error δ_{ideal}

The state of skill mastery maps onto Ψ and the gap gap through Bayesian knowledge tracing [8] and its neural-network development [9], which give a probabilistic estimate of mastery from the learner's answer history. The form of the anchor B accords with self-determination theory: focus, charge, and integration resonate with the basic needs of autonomy, competence, and relatedness [10]. Removing the evaluative frame lowers σ and thereby narrows the ideal error [5].

Retrieval practice strengthens the state Ψ through active recall [11], while it sets no topology of the difficulty control parameter and so remains peripheral relative to the spine of the invariant. Learning analytics [12] supplies data for the proxy metrics of Section 7, while it carries no interior-optimum topology of its own. The peripheral status is held with the status of **HYPOTHESIS**: both approaches enter the applied practice of the dyad while remaining outside the formal spine.

Analogy, not identity. All correspondences of Tables 2 and 3 are structural analogies at the level of control-parameter topology, while the psychometric and operator

Table 3: Peripheral approaches: practical value outside the formal spine of the invariant.

Approach	Anchor	Why off-spine
Retrieval practice	Testing as learning	Strengthens Ψ, sets no topology of ρ
Learning analytics	Trajectory data	Feeds proxy metrics, with no optimum topology of its own

quantities differ; no identification of quantities is made here.

9 Conclusion: predictions, boundaries, and a falsifiable program

A single structural invariant is confirmed: the learner and the AI tutor form a coherent dyad whose state converges to a self-consistent fixed point of competence with an interior optimum of difficulty, and the maximum appearance of mastery is the signature of the ideal error,

$$\Psi^* = \Phi(\Psi^*), \qquad 0 < \rho^* < 1, \tag{12}$$

$$\text{Quality}(\rho) \text{ single-peaked}; \qquad \max \text{Appearance} = \text{signature}(\delta_{\text{ideal}}). \tag{13}$$

Epistemic stratification. The final fixed point (12) and the signature of the maximum appearance (13) gather the result of the work. Declared as structural invariants (**L2-INVARIANT**) are the weak-link property (4), the existence of the interior optimum of difficulty (5), and the existence of the interior optimum of reliability (11). Declared as predictions are the eight empirically testable consequences listed below. Declared as hypotheses are the fixed-point operator Φ (2), the numerical values of ρ^* and S^*, and the metrics of Section 7 at the level of measurable quantities [3].

Nine falsifiable tests (eight predictions, one hypothesis). The program is operationalizable and falsifiable; Table 4 collects each prediction, its applied observable, the falsifier, and the level.

P1 (inverted U of quality vs difficulty). Learning quality as a function of presentation difficulty ρ is single-peaked with an interior maximum $\rho^* \in (0, 1)$: as $\rho \to 0$ boredom and stagnation, as $\rho \to 1$ overload and coherence breakdown. A monotone rise or a monotone fall of quality in ρ falsifies the existence of the interior optimum (**PREDICTION**, the sharpest).

P2 (presentation at ρ^* accelerates gap reduction). Holding the presentation in the neighbourhood of ρ^* accelerates the reduction of the mastery gap gap relative to

Table 4: Validation program: mapping of predictions P1–P9 onto applied observables (tests to be run; no correspondence is declared confirmed).

Pred.	Applied observable	Falsifier	Level
P1	Learning quality vs difficulty ρ	Monotonicity instead of a peak	PREDICTION
P2	Reduction of gap under presentation near ρ^*	No acceleration at ρ^*	PREDICTION
P3	Weak link of B predicts session breakdown	Breakdown at a high smallest factor	PREDICTION
P4	Confidence vs test result	Joint growth of both	PREDICTION
P5	Closed reflection loop vs broken loop	Superiority of the broken loop	PREDICTION
P6	Calibration of self-assessment vs mastery	Accurate calibration at a large gap	PREDICTION
P7	Mastery gain vs the share of reliance S	Monotonicity instead of a peak S^*	PREDICTION
P8	Removing the evaluative frame lowers σ	No reduction of the mismatch	PREDICTION
P9	Order effects in dyad self-assessment	No order effects	HYPOTHESIS

a fixed difficulty; the absence of acceleration at ρ^* falsifies the role of the interior optimum (**PREDICTION**).

P3 (the weak link predicts breakdown). The smallest of the factors F, E, $(1-\sigma)$, Λ predicts a learning-session breakdown better than their average; a session breakdown at a high smallest factor falsifies the weak-link property (4) (**PREDICTION**).

P4 (appearance of mastery runs ahead of mastery). The learner's subjective confidence after a coherent explanation exceeds the result of a delayed test without hints; a joint growth of confidence and test result falsifies the identification of the maximum appearance with the signature of the ideal error (**PREDICTION**).

P5 (the closed reflection loop beats the broken one). Dyads with a closed reflection phase (10) reduce the mastery gap faster than dyads without reflection at equal time; a superiority of the broken loop falsifies the role of self-regulation (**PREDICTION**).

P6 (calibration of self-assessment diverges from mastery). The calibration of the learner's self-assessment diverges from factual mastery the more strongly the higher the coherence of the presentation at a large gap gap; an accurate calibration at a large gap falsifies the ideal-error mechanism (**PREDICTION**).

P7 (interior optimum of reliance). The mastery gain as a function of the share of reliance S on the tutor is single-peaked with an interior maximum $S^* \in (0,1)$: both too low and too high a reliance lower the gain; a monotone dependence of the gain on S falsifies the existence of the optimum of reliability (11) (**PREDICTION**).

P8 (removing the evaluative frame lowers the mismatch). Removing the evaluative frame and stereotype threat measurably lowers the mismatch σ and raises the observed result while competence is preserved; the absence of a reduction of σ falsifies the channel of the confidence factor (**PREDICTION**).

P9 (order effects in dyad self-assessment). The joint self-assessment of the learner and the tutor exhibits order effects of question presentation, described by a context-dependent model better than the classical one at an equal number of parameters; the absence of order effects falsifies the contextual reading of dyad self-observation (**HYPOTHESIS**, the boldest).

Boundaries and risk. The boundaries are fixed honestly. The cross-domain correspondences remain structural analogies of control-parameter topology; the psychometric and operator quantities differ, and a direct identification of one scalar across domains slides into an imitation of resonance [3]. The operator Φ and the numerical values of ρ^*, S^* are held at the level of hypothesis; the metrics are operationalizable proxies subject to the same program of falsification. The eight predictions are put forward as an operationalizable agenda for the empirical testing of the learning dyad.

CONFLICT OF INTEREST

The author declares no conflict of interest.

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