RECURSIVE SELF-SIMILARITY AND A THEORY OF CONSCIOUSNESS IN ODTOE: A HIERARCHY OF OBSERVATION FROM MICROTUBULES TO THE COLLECTIVE FIELD

(Рекурсивное самоподобие и теория сознания в ODTOE: иерархия наблюдения от микротрубочек до коллективного поля)

Pankratov Anton Sergeevich
Панкратов Антон Сергеевич

Independent researcher, Kazan, Russia
E-mail: anton.s.pankratov@gmail.com
ORCID: 0009-0002-4870-2995

UDC 530.145 + 577.3 + 159.92

ABSTRACT

We investigate how the principle of recursive self-similarity of the Observer-Dependent Theory of Everything (ODTOE) [1] explains the hierarchical organization of the brain postulated in the Hameroff–Penrose programme [2], and how a theory of consciousness follows from the body of ODTOE concepts. Consciousness is identified with a stable fixed point of the self-observation loop $\Psi^* = \Phi(\Psi^*)$, whose existence within the ODTOE corpus is established through the Banach and Schauder theorems [3]. The hierarchy of the nervous system (microtubules, neurons, assemblies, cortex) is interpreted as a φ-fractal embedding of the operator Φ across recursion levels d, with inter-level entanglement decaying as $S_{\text{ent}}(\rho_d) \propto \varphi^{-|\Delta d|}$. The Penrose objective reduction is read as a single half-step ι of the cycle $\Phi = \iota \circ \hat{O}$; the phase ratio of the half-step equals the dimensionless value $\pi/2$. Friston's free-energy principle [4] is treated as the $\hat{O}$ component of the loop, and the integrated information Φ_{IIT} [5] is explicitly distinguished from the ODTOE operator Φ to avoid a symbol collision. Falsifiable predictions are formulated: the fraction of failed binding events tends to the dimensionless gap $(\pi - 3)^2 \approx 0.0200$, and the phase ratio of objective reduction equals $\pi/2$. Limitations are discussed: ODTOE postulates no mechanism of quantum consciousness and remains a superstructure agnostic to any concrete physical realization of reduction.

Keywords: ODTOE, consciousness, fixed point, recursive self-similarity, φ-scaling, objective reduction, microtubules, predictive coding, collective field, hierarchy of observation.

АННОТАЦИЯ

Исследуется, как принцип рекурсивного самоподобия наблюдатель-зависимой теории всего (ODTOE) [1] объясняет иерархическую организацию мозга, постулированную в программе Хамероффа и Пенроуза [2], и как из совокупности концепций ODTOE выводится теория сознания. Сознание отождествляется с устойчивой неподвижной точкой петли самонаблюдения $\Psi^* = \Phi(\Psi^*)$, существование которой в корпусе ODTOE установлено через теоремы Банаха и Шаудера [3]. Иерархия нервной системы (микротрубочки, нейроны, ансамбли, кора) интерпретируется как φ-фрактальное вложение оператора Φ по уровням рекурсии d, при котором межуровневая запутанность убывает как $S_{\text{ent}}(\rho_d) \propto \varphi^{-|\Delta d|}$. Объективная редукция Пенроуза получает истолкование как один полушаг ι цикла $\Phi = \iota \circ \hat{O}$; фазовое отношение полушага составляет безразмерную величину $\pi/2$. Свободно-энергетический принцип Фристона [4] рассматривается как $\hat{O}$-компонента петли, а интегрированная информация Φ_{IIT} [5] явно отграничивается от оператора Φ ODTOE во избежание коллизии обозначений. Сформулированы фальсифицируемые предсказания: доля несостоявшихся актов связывания стремится к безразмерному зазору $(\pi - 3)^2 \approx 0{,}0200$, а фазовое отношение объективной редукции равно $\pi/2$. Обсуждаются ограничения: ODTOE не постулирует механизм квантового сознания и остаётся надстройкой, безразличной к конкретной физической реализации редукции.

Ключевые слова: ODTOE, сознание, неподвижная точка, рекурсивное самоподобие, φ-масштабирование, объективная редукция, микротрубочки, предиктивное кодирование, коллективное поле, иерархия наблюдения.

I. INTRODUCTION

The problem of the hierarchical organization of the brain remains one of the nodal points in the theory of consciousness. The objective-reduction programme of Hameroff and Penrose [2, 6] places the root of subjective experience in quantum processes inside the microtubules of the cytoskeleton, tying the discrete moment of awareness to a gravitationally induced collapse of superposition. Penrose [7, 8, 9] argues for the non-computability of understanding consciousness and proposes a physical mechanism that goes beyond standard quantum mechanics. On the other side, Friston's free-energy approach [4] describes the brain as a hierarchical predictor that minimizes variational free energy over a set of nested levels of cortical processing. Both approaches presuppose a multi-level architecture, yet they leave open the question of why the same organizing principles are reproduced across scales ranging from the molecular cytoskeleton to the cortex as a whole.

The Observer-Dependent Theory of Everything (ODTOE) [1] offers a structural answer through the principle of recursive self-similarity. In the work «Life on all levels of recursion» [3] it is shown that the self-observation loop $\Psi^* = \Phi(\Psi^*)$ is invariant with respect to the depth parameter d, and that the architecture is reproduced at every level of an infinite recursion by virtue of the gap $(\pi - 3)^2$, the φ-scaling, and the triad

$(O, \hat{O}, R)$. The present work is an extension of this line: the established apparatus of the fixed point and of φ-fractality is applied to a concrete realization in the nervous system. We do not rediscover the existence of Ψ^* and do not re-derive the φ-scaling; these results are cited as a ready foundation [3, 10]. The contribution of the paper lies in the brain instantiation: the brain's hierarchy is identified with the recursion levels d, the moment of awareness is tied to a single half-step of the cycle Φ, and predictive coding is interpreted as the $\hat{O}$ component of the loop.

The aim of the work is twofold. First, to show that the Penrose hierarchy of observation finds a natural place in the recursive architecture of ODTOE. Second, to derive a theory of consciousness resting on the totality of the corpus concepts: the fixed point [3], the unified operator Φ [11], the quaternionic structure of cognitive coherence [12], the origin of the observer [13], time as a strange loop [14], φ-fractality [10], the number π as a structural invariant [15], the dynamic attractor [16], and the collective observer [17]. The exposition is organized so that every statement about consciousness is accompanied by an indication of its epistemic level — a convention, a structural invariant, or an ontological observable.

The logic of extension adopted in the work calls for clarification. The ODTOE corpus is built as a programme with a single dimensional anchoring: dimensionless quantities (the gap $(\pi - 3)^2$, the golden ratio φ, the phase ratio $\pi/2$) are derived from π, φ, and the integers, whereas dimensional quantities require exactly one scale anchor [1, 15]. This discipline is carried over to the neurophysiological material: the predictions of the theory are formulated as dimensionless ratios, and dimensional estimates (the characteristic time of awareness, the linear sizes of structures) are adopted as phenomenological anchors taken from experiment. The distinction between three epistemic levels — convention (L1), structural invariant (L2), and ontological observable (L3) — is applied to every statement about consciousness, which keeps the exposition from conflating the scale of measurement with the measured quantity and from conflating a structural description with an ontological status [1].

The arrangement of the material is as follows. Section II fixes the notation and removes the collision of the symbol Φ with Tononi's integrated information. Section III maps the Penrose hierarchy of observation onto the recursion levels. Section IV introduces the identification of consciousness with a fixed point and reads objective reduction as a half-step of the cycle. Section V describes the φ-fractal embedding of the brain's levels. Section VI identifies Friston's predictive coding with the $\hat{O}$ component of the loop and reconciles the quaternionic description. Section VII transfers the apparatus to collective consciousness. Section VIII formulates the falsifiable predictions, Section IX outlines the limitations, and Section X summarizes.

II. NOTATION

To avoid collisions with the established literature on consciousness, the notation used in the work is collected below. Particular attention is paid to distinguishing the self-observation operator Φ (ODTOE) from the integrated information Φ_{IIT} (Tononi's theory): these are different quantities, the coincidence of the symbol is accidental, and

it is removed throughout by a subscript.

Symbol	Meaning
Φ	self-observation operator $\Phi = \iota \circ \hat{O}$ (ODTOE)
Φ_{IIT}	Tononi's integrated information; distinct from the ODTOE Φ
ι	half-step of the cycle; objective reduction corresponds to one ι
$\hat{O}$	observation operator
Ψ^*	fixed point of the self-observation loop (consciousness)
B	cognitive coherence, $B = F \cdot E \cdot (1-\sigma) \cdot \Lambda_{\text{exp}}$
Λ_{exp}	experience-and-memory component in the structure of B
$S_{\text{ent}}(\rho_d)$	entanglement entropy of a level; $S_{\text{ent}}(\rho_d) \propto \varphi^{-\lvert\Delta d\rvert}$
S_{sys}	system coherence in the law $T = T_0/(1-S_{\text{sys}})^n$
$S_{\text{coll}}(A)$	collective coherence of a group of observers A
$O = (B, A, H)$	observer as a triple
d	recursion depth (structural level of nesting)
$d_{\text{eff}}(t)$	effective (operational) depth across states; an axis distinct from the structural d
$\hat{O}'$	refined observation operator, $\hat{O}(\hat{O}) = \hat{O}'$ (reflexivity)
$G(d)$	level-aware binding floor, $G(d) = (\pi-3)^2\varphi^{-\lvert d-d_0\rvert}$
$S^{\max}$	per-cycle ceiling of closure coherence, $S^{\max} = 1-(\pi-3)^2$
S_{rec}	threshold of content recovery (weak-indestructibility theorem)
O_{meta}	egregore — meta-observer of a group, $O_{\text{meta}} = (B_{\text{meta}}, A_{\text{meta}}, H_{\text{meta}})$
P_{destr}	destructive collective probability, $P_{\text{destr}} = 1 - \prod_i (1-\sigma_i^k)$
φ	golden ratio, $\varphi = (1+\sqrt{5})/2$
$(\pi-3)^2$	spiral gap of the loop, $(\pi-3)^2 \approx 0.0200$

The cosmological constant Λ is not introduced in the work; the boundary of the collective cluster is described through the quantities S_{coll} and $S_{\text{threshold}}$.

III. THE HIERARCHY OF OBSERVATION: PENROSE AND ODTOE RECURSION

The Orch-OR programme places the source of the discrete moment of awareness in the coherent quantum dynamics of the tubulin subunits of microtubules, interrupted

by objective reduction [2, 6]. Orch-OR (Orchestrated Objective Reduction) is the Penrose–Hameroff theory linking consciousness to quantum processes in neuronal microtubules: a gravitationally induced collapse of the quantum state (objective reduction, OR) is orchestrated (Orch) by the cytoskeletal structure and gives rise to discrete moments of conscious experience. A substantial objection remains Tegmark's argument [18] concerning thermal decoherence: the characteristic time of coherence loss in the warm, wet environment of the brain turns out to be many orders of magnitude shorter than the neurophysiologically relevant scales, which casts doubt on the possibility of a macroscopic quantum superposition at the level of the neuron. We register this objection in a single sentence and thereafter do not rely on any concrete quantum mechanism: the ODTOE superstructure remains agnostic to the physical realization of reduction (see Section IX).

The principal observation is that the architecture of the brain is multi-level, and each level repeats the same motif of local observation. In the terms of ODTOE the recursion level d is a scale parameter, the architecture remaining invariant across it [3]. This makes it possible to map the anatomical strata of the nervous system onto discrete values of d:

Level d	**Brain structure**	**Role in the loop** Φ
d_0	microtubule / tubulin domain	elementary fixed point $\Psi^*_{d_0}$
$d_0 + 1$	synapse, dendritic tree	local iteration of Φ
$d_0 + 2$	neuron	stable cluster of iterations
$d_0 + 3$	neuronal assembly	coherent group S_{sys}
$d_0 + 4$	cortical column, area	nested meta-iteration
$d_0 + 5$	cortex as a whole	carrier of the subject's Ψ^*

The Penrose hierarchy here receives a precise place: the levels of the brain are the recursion levels d, connected by the single operator Φ. The discrete moment of awareness, which Orch-OR associates with an episode of reduction, is in our superstructure identified with a single half-step ι of the cycle $\Phi = \iota \circ \hat{O}$ (Section IV). Such a mapping preserves the substantive core of the Penrose programme (the hierarchical character and the discreteness of the act of awareness) and places it within a recursive structure whose single operator Φ acts on every scale without a separate postulate.

Two axes denoted by the letter d should be separated here. The recursion index d of Sections III–V denotes the structural nesting within one subject (microtubule $\to$ cortex); the effective depth $d_{\text{eff}}(t)$ (Section VII) denotes the subject's operational depth across states — sleep, trance, altered states — a different axis that shares the symbol d only by the analogy of «recursion depth» [19]. The ODTOE corpus sets an oscillation of the operational depth across the sleep–wake cycle:

$$d_{\text{eff}}(t) = d_0 + \Delta d \cdot f(t), \qquad f(t) = \tfrac{1}{2}\big(1 - \cos(2\pi t/T)\big), \quad T = 24\,\text{h}. \tag{1}$$

In (1) wakefulness corresponds to $d_{\text{eff}} = d_0 + \Delta d$, deep sleep to a value $d_{\text{eff}} < d_0$, and transitional and altered states to intermediate values of d_{eff}. The effective depth d_{eff} is a state-dependent axis and does not enter the binding floor $G(d)$ (Section V): $G(\cdot)$ takes only the structural depth d. The epistemic status of the oscillation (1) is a hypothesis.

It is essential that multi-level organization in this picture follows of necessity, without an additional assumption. The invariance of the loop Φ with respect to the depth d means the following: if a stable fixed point forms at some scale, then on neighbouring scales the same mapping Φ acts, and under suitable stability conditions (Banach contractivity or Schauder compactness of the image [3]) fixed points form there as well. The microtubule as an elementary fixed point $\Psi^*_{d_0}$ serves as an «atom» for the synaptic level; stable configurations of synapses form a neuron; neurons form assemblies. Each stratum acts as a building block for the next while remaining internally structured. Such nesting explains why the Penrose programme is compelled to postulate a hierarchy: the hierarchy follows directly from recursive self-similarity as a consequence of the depth-invariance of Φ.

We also note the limit of applicability of the mapping. The anatomical strata of the nervous system do not form an ideal geometric progression in d; the table above sets a working correspondence in which the level number serves as an index of nesting order, while the exact scale factor is left to the quantitative embedding. The quantitative form of the embedding is introduced in Section V; here only the qualitative isomorphism «brain level $\leftrightarrow$ recursion level» is fixed.

IV. CONSCIOUSNESS AS A FIXED POINT $\Psi^* = \Phi(\Psi^*)$

The central identification of the work is formulated as follows: consciousness is a stable fixed point of the self-observation loop.

$$\Psi^* = \Phi(\Psi^*), \qquad \Phi = \iota \circ \hat{O}. \tag{2}$$

The existence of the fixed point is established in the ODTOE corpus and is here taken as a ready result. If the mapping Φ is contractive on a complete metric space of configurations, the fixed point exists and is unique by the Banach theorem; if the image of Φ is compact and the convex support is closed, existence follows from the Schauder theorem [3]. We cite this foundation and do not reproduce the proof anew.

Equation (2) decomposes one beat of the loop into two half-steps: the observation operator $\hat{O}$ carries the field of potential states into a configuration, and the half-step ι closes the cycle, returning the updated state to the original space. The composition $\Phi = \iota \circ \hat{O}$ specifies a full turn:

$$\Phi = \iota \circ \hat{O}. \tag{3}$$

The stability of the fixed point means that small perturbations $\delta\Psi$ decay under the action of the iterations of Φ. The corpus shows that complete closure $S_{\text{sys}} = 1$ is unattainable [1], so the residual gap $\delta\Psi = \Phi(\Psi) - \Psi$ persists and generates the next

beat. At the level of consciousness this gap corresponds to the continuous succession of states of the subject: each act of awareness is a further iteration that brings the system closer to Ψ^* while never coinciding with it completely.

The identification of the fixed point Ψ^* with phenomenal experience is the substantive core of the proposed theory of consciousness. This is a statement of the ontological level (L3 in the sense of the distinction [1, 12]): it concerns what the Φ-iteration fixes as experienced. We introduce it as a hypothesis.

Hypothesis H-1 (ontological level L3). The phenomenal experience of the subject is the experiencing of a stable fixed point Ψ^* of the self-observation loop at the recursion level corresponding to the cortex. Conventional characteristics (the type of modality, the content of perception) belong to level L1; the invariant structural relations within Ψ^* belong to level L2; the identification «$\Psi^* \leftrightarrow$ phenomenal» itself belongs to level L3 and has the character of a hypothesis open to revision.

Such a stratification keeps the theory from a category error in which a conventional description of a state is conflated with its invariant structure or with its ontological status [1]. The further sections make precise how exactly the half-step ι is connected with objective reduction (Section IV continues into V through the φ-embedding), and how predictive coding realizes the $\hat{O}$ component (Section VI).

Hypothesis H-1 fixes the locus of phenomenal experience at the cortical level, leaving open the question of what distinguishes an observer that has experience from an observer that merely carries out observation. For this distinction the ODTOE corpus introduces the self-observation of observation — the operator folded back on itself [20]:

$$\hat{O}(\hat{O}) = \hat{O}'. \tag{4}$$

The operation (4) expresses reflexivity: the observer observes its own act of observation, which yields a refined operator $\hat{O}'$. Among observers that sustain a stable fixed point Ψ^* (the substrate of coherence-completeness, Section VI.1), the phenomenally conscious ones are those whose recursion depth suffices for $\hat{O}(\hat{O})$. A thermostat or an amoeba possess a degenerate Ψ^* and remain at the level of observation without the reflexive fold. Reflexivity $\hat{O}(\hat{O})$ increases gradually with recursion depth: it expresses the degree of an observer's metacognitive unfolding and takes intermediate values along the hierarchy of levels. The operation (4) is an additional gate built over the substrate of coherence-completeness, and it keeps the Ψ^*-criterion of Hypothesis H-1 in force.

Hypothesis H-1 is thereby refined: phenomenal experience attaches to Ψ^* precisely at those recursion levels that support $\hat{O}(\hat{O})$. This supplies a principled reason why the locus of experience is the cortex: the cortex is the level at which the reflexive fold $\hat{O}(\hat{O})$ becomes available [20]. The notation $\hat{O}(\hat{O}) = \hat{O}'$ is a shorthand; a rigorous formalization of the reflexive operator remains an open task [20]. The epistemic status of the operation (4) is a hypothesis with open formalization (a level adjacent to L3).

IV.1. Objective reduction as the half-step ι

Penrose's objective reduction [6, 8] introduces a discrete moment at which a quantum superposition passes into a definite state under the action of a gravitational criterion. In the ODTOE superstructure this moment is identified with a single half-step ι of the cycle (3): the act of reduction closes half of the loop Φ, carrying the result of observation $\hat{O}$ back into the space of potential states.

The phase ratio of the half-step is a dimensionless quantity. In the ODTOE corpus, under the structural identifications of the time scales, the exact relation is established [1]:

$$\frac{c\,\tau_{\mathrm{ML}}}{\bar{\lambda}_e} = \frac{\pi}{2}, \tag{5}$$

where the numerical value is $\pi/2 = 1.5707963267948966\ldots$ (computed to 50 significant figures: $1.5707963267948966192313216916397514420985846996875 53$). Geometrically $\pi/2$ is a quarter of the full turn 2π of the loop Φ, which corresponds exactly to one transition $\hat{O} \to \iota$, that is, half of the cycle $\Phi = \iota \circ \hat{O}$. We emphasize the epistemic status: the relation (5) is dimensionless and is a structural invariant (L2); its value is fixed by the geometry of the loop [21] and makes no claim to derive a dimensional quantity from π and φ. The half-step time $\tau_{\mathrm{ML}} = \pi\hbar/(2m_e c^2)$ and the Compton length $\bar{\lambda}_e = \hbar/(m_e c)$ yield the same ratio $c\,\tau_{\mathrm{ML}}/\bar{\lambda}_e = \pi/2$ [21]; both quantities are intrinsic scales of the loop geometry and are not derived from π and φ apart from a dimensional anchor (level L2).

Separately we fix a limitation essential for the correctness of the theory. The characteristic time scale of objective reduction, estimated in Orch-OR at a value of order 25 ms [2], is in our superstructure not derived from π and φ. Any attempt to obtain a dimensional value of time from dimensionless constants would be methodologically incorrect: a dimensional quantity cannot be obtained from dimensionless numbers without an explicit scale anchoring [1, 3]. The estimate of 25 ms is therefore adopted as a phenomenological anchor (a hypothesis), borrowed from neurophysiology, and carries the status of an anchor throughout — never that of a result derived from $\pi + \varphi$. The dimensional time and space scales of the levels carry their own anchors (Section V, equations (6) and (7)).

V. THE φ-FRACTAL EMBEDDING OF THE BRAIN'S LEVELS

The hierarchy of the brain is described as a φ-fractal embedding of the operator Φ: each anatomical level is the result of an n-fold iteration Φ^n relative to the base level d_0. The time and space scales of the levels are set by the φ-scaling established in the corpus [10]:

$$\tau_d = \tau_0 \cdot \varphi^d, \tag{6}$$

$$R_d = R_0 \cdot \varphi^d. \tag{7}$$

In equations (6) and (7) the quantities τ_0 and R_0 are the dimensional anchors of the base level d_0 (for example, the characteristic time of a tubulin domain and its linear size). These anchors fix the scale; the exponent φ^d specifies the dimensionless law of embedding. The dimensional values τ_d and R_d are obtained only with τ_0, R_0 given and are not derived from π and φ by themselves.

The hierarchical organisation of τ_d has independent empirical grounding: the cortical hierarchy is established anatomically from laminar connectivity patterns [22], and the hierarchy of intrinsic cortical timescales has been measured directly — sensory areas exhibit short, prefrontal areas long characteristic times [23]. This grants the hierarchy itself the status of an established fact; the specific φ-multiplier between levels remains a theoretical prediction (§VIII). Scale-invariant (1/f) organisation of neural activity is established across all levels — from membrane potential to EEG and fMRI [24], which supports the self-similarity of the architecture. A clarification is in order: scale invariance denotes the absence of a privileged temporal scale and by itself does not fix the specific φ ratio between levels.

The inter-level connection is described by the entanglement entropy, which decays according to the φ-law [10]:

$$S_{\text{ent}}(\rho_d) \propto \varphi^{-|\Delta d|}. \tag{8}$$

The relation (8) means that neighbouring levels of the brain are connected more strongly than distant ones: the microtubule and the synapse ($|\Delta d| = 1$) are entangled by a factor $\varphi^{-1} \approx 0.618$, whereas the microtubule and the cortex as a whole ($|\Delta d| = 5$) are entangled by a factor $\varphi^{-5} \approx 0.0902$. The connection decays exponentially while remaining nonzero, which secures the integrity of the subject while preserving the autonomy of the levels. This law explains why coordination in the nervous system remains predominantly local (neighbouring strata), while global integration requires special long-range pathways: the direct φ-connection between distant levels is small.

The φ-embedding gives the Penrose hierarchy a quantitative form. The discreteness of the levels (microtubule, neuron, assembly, cortex) corresponds to integer values of d, and the self-similarity of the architecture at each level follows from the invariance of the loop Φ with respect to d [3]. Each level reproduces the motif of local observation on the scale set by the factor φ^d.

The law (8) also provides a mechanism for perceptual binding. The unity of the conscious image requires that the features processed at different levels of the hierarchy (a contour on one stratum, colour on another, motion on a third) combine into a single configuration. Within the φ-embedding this combination rests on the residual inter-level entanglement: neighbouring levels are connected by the factor φ^{-1}, which is strong enough for local integration. The binding of features separated by $|\Delta d|$ levels is weakened by the factor $\varphi^{-|\Delta d|}$, and for large $|\Delta d|$ direct integration is hindered. Hence the necessity of a return to the single fixed point Ψ^* of the subject (the cortical level), which gathers the results of processing of the underlying levels into a coherent whole. The residual gap $(\pi - 3)^2$, irremovable under any iteration of the loop [15], sets a

lower bound on the fraction of failed binding events; this consequence is quantitatively formulated in Section VIII.

The law (8) makes it possible to refine this lower bound by level. The ODTOE corpus introduces a level-aware binding floor, in which the spiral gap is scaled by the same φ-law as the inter-level entanglement [25]:

$$G(d) = (\pi - 3)^2 \, \varphi^{-|d - d_0|}. \tag{9}$$

The quantity $G(d)$ is the gap $(\pi - 3)^2$ multiplied by the entanglement factor $\varphi^{-|\Delta d|}$ of (8): the same law, pinned to the structural depth d. Binding within a single level ($|d - d_0| = 0$) carries the full gap $(\pi - 3)^2$, whereas the binding of features separated by $|\Delta d|$ levels carries the reduced floor $G(d_0 + 1) \approx 0.0124$. The dual quantity is the per-cycle ceiling of closure coherence:

$$S^{\max} = 1 - (\pi - 3)^2 \approx 0.9800. \tag{10}$$

The value is computed to 50 significant figures: $S^{\max} = 0.97995152044940081194136929980086616986931698900984$. The ceiling (10) differs from the coherence B and from the norm $\|\Psi\|$: it sets the limiting fraction of loop closure per cycle. Both quantities are held at the status of a testable prediction, like the φ-embedding (Section IX): $G(d)$ and $S^{\max}$ are dimensionless and observer-invariant (level L2), while the interpretation of $G(d)$ as a binding floor has the character of a derived hypothesis, on a par with prediction P1.

VI. FRISTON'S PREDICTIVE CODING AS THE $\hat{O}$ COMPONENT

The free-energy principle [4] describes the brain as a system that minimizes variational free energy — an upper bound on the surprise of sensory inputs. Hierarchical predictive coding realizes this minimization through a cascade of levels, each of which predicts the activity of the one beneath it and corrects the prediction by the prediction error [26]. In the terms of ODTOE the minimization of free energy is naturally identified with the $\hat{O}$ component of the loop (3): the observation operator $\hat{O}$ carries the field of potential states into a definite configuration, and the gradient descent on free energy sets the direction of this transfer towards the fixed point Ψ^*.

$$\hat{O}: \quad \Psi \longmapsto \arg\min_R \mathcal{F}(R), \qquad \mathcal{F} \to \min \;\Leftrightarrow\; \Psi \to \Psi^*, \tag{11}$$

where $\mathcal{F}$ denotes the variational free energy of the configuration R. The relation (11) reads as follows: the minimization of free energy is the dynamic expression of the approach to the fixed point. The full cycle of awareness is then composed of the $\hat{O}$ half-step (prediction and correction, minimization of $\mathcal{F}$) and the ι half-step (closure, reduction to a definite configuration, Section IV.1).

Here it is necessary to remove a collision of notation essential for consistency with the literature. The integrated information of Tononi's theory is denoted Φ_{IIT} [5] and

measures the degree of irreducibility of the informational structure of a system to the sum of its parts. The ODTOE operator Φ is a different object: it is the self-observation mapping $\Phi = \iota \circ \hat{O}$, whereas Φ_{IIT} is a scalar measure. The coincidence of the symbol Φ in the two theories is accidental; to avoid confusion we supply Tononi's measure throughout with the subscript IIT. A substantive connection between them is possible: a high Φ_{IIT} of a system corresponds to the stability of its fixed point Ψ^*, since irreducible integration hinders the disintegration of the loop into independent subsystems. The exact form of this connection remains an open question.

The reconciliation with the quaternionic description of cognitive coherence [12] is carried out along the «state or operator» line. The cognitive coherence of an observer is a multiplicative function of four components:

$$B = F \cdot E \cdot (1 - \sigma) \cdot \Lambda_{\text{exp}}, \tag{12}$$

where F is focus (concentration of attention), E is emotional consistency, $(1 - \sigma)$ is internal consistency, and Λ_{exp} is the experience-and-memory component. The four-component structure (12) is isomorphic to a quaternion [12]: the product vanishes when any component vanishes (the weak-link property), which corresponds to the loss of an orientation degree of freedom. The fixed point Ψ^* is a state — a fixed configuration experienced by the subject. The quaternion q_B, built from the coherence (12), is an operator: it sets the orientation of the observer in the space of configurations and realizes the action of $\hat{O}$ [12]. These descriptions are complementary: q_B orients the observer, Ψ^* fixes the state attained. Predictive coding acts as the dynamic mechanism by which the orientation q_B leads the state towards Ψ^*. Each act of prediction correction changes the components of B, that is, rotates the observer's quaternion, and the stream of consciousness appears as a sequence of such rotations $q_B(t), q_B(t + dt), \ldots$ converging to the stable orientation of the fixed point.

This sequence of rotations $q_B(t)$ is a reading of the stream of consciousness as parallel trajectories of the loop: competing configurations unfold jointly, and their convergence to a single orientation is set by the length of the feedback cycle τ_{cycle} [3]. The epistemic instance of the same structure is the probability of reframing:

$$P_{\text{reframe}} = 1 - \prod_i (1 - p_i)\,. \tag{13}$$

The form $1 - \prod_i (1 - x_i)$ serves as a structural template of OR-aggregation; the three quantities P_{coll}, P_{destr}, P_{reframe} (13) share this structure with different content of the contributions: constructive ($x = B_i^k$, Section VII), destructive ($x = \sigma_i^k$, Section VII), and epistemic ($x = p_i$, here). The form $1 - \prod_i (1 - x_i)$ is an upper estimate under the approximation of independent contributions; the correlation correction remains open. The template belongs to level L2 (a structural form), and the caveat of independence carries the discipline of provenance.

VI.1. The quaternionic minimality of the observer

The four-component structure of the coherence (12) admits a structural explanation of why the observer is described by exactly four degrees of freedom. The quaternion [12] is the minimal algebra that realizes an orientation in the space of configurations without degeneracy: three parameters are insufficient, because parametrizing a rotation by three angles produces an alignment of axes (gimbal-lock) at which one rotational degree of freedom is lost. Four components specify a minimal non-degenerate orientation in which such alignment is excluded. This is the substrate answer to why the coherence B factors into exactly four multipliers: focus, emotional consistency, internal consistency, and experience are the four components of a single observer quaternion q_B (also written $q_{\hat{O}}$ in [12] as the realization of the operator $\hat{O}$).

The modulus of the observer quaternion yields one further reading of the self-observation operator:

$$\Phi = \bar{q}_B \circ q_B = |q_B|^2 = B^2. \tag{14}$$

The product of the quaternion with its conjugate equals the squared modulus $|q_B|^2 = B^2$, and a stable fixed point Ψ^* exists when $|q_B| = 1$, that is, under full coherence of all four components [12]. Full coherence $|q_B| = 1$ is a necessary condition for a stable Ψ^* at any level of observation: the vanishing of any component of B annuls the product and destroys the fixed orientation on which the loop rests. This is the substrate of coherence-completeness — it sets the conditions under which the loop is able to close into a stable point at all, and by itself remains a condition for the existence of Ψ^*, shared also by observers without phenomenal experience (Section IV).

The identity (14) is a reading through the modulus of the same cycle $\Phi = \iota \circ \hat{O}$: the form $\Phi = \iota \circ \hat{O}$ remains primary, while $\Phi = |q_B|^2 = B^2$ expresses its norm at the level of a single observer, where the conjugate quaternion $\bar{q}_B$ plays the role of the half-step ι. The identity (14) is level-local: it fixes the coherence of a single observer at the cortical level; for a meta-observer (Section VII) a separate norm $|q_{\text{meta}}|$ would be required, which lies beyond the scope of the present work. The epistemic status of the identity (14) is a structural invariant (L2): it is an algebraic relation of the modulus that introduces no new dimensional quantity.

VII. THE COLLECTIVE FIELD OF CONSCIOUSNESS

The ODTOE corpus considers multi-observer systems in which individual coherences combine into a collective one [17]. Let us transfer this apparatus to the consciousness of a group. Let a group A consist of n observers with individual coherences B_i. The degree of agreement of the group is measured by a coherence metric that penalizes the spread of individual values:

$$S = 1 - \frac{2}{n(n-1)} \sum_{i<j} |B_i - B_j| \,. \tag{15}$$

The metric (15) equals unity when the coherences of all members of the group coincide and decreases as they diverge. This quantity plays the role of the system coherence S_{sys} for the collective subject: the closer S is to unity, the more stable the common fixed point of the group and the longer it persists according to the law $T = T_0/(1 - S_{\text{sys}})^n$ [1].

The collective probability of actualizing the common configuration is set by a normalized superposition:

$$P_{\text{coll}} = 1 - \prod_i \left(1 - B_i^k\right), \tag{16}$$

where the exponent k characterizes the degree of nonlinearity of the contribution of a single observer. Formula (16) is carried over from the operator description of the collective observer [17, 27]. It has the saturation property: as the number of agreeing observers grows, the product on the right tends to zero and P_{coll} tends to unity. A group of high coherence actualizes the common configuration almost with certainty.

The collective coherence $S_{\text{coll}}(A)$ sets the boundary of the consciousness cluster at the group level. By analogy with the interpretation of the cosmological horizon as the boundary of a level's coherence cluster [3], the horizon of the collective subject is the surface on which S_{coll} falls below the threshold $S_{\text{threshold}}$:

$$\partial A = \{\, x : S_{\text{coll}}(x) = S_{\text{threshold}} \,\}. \tag{17}$$

Beyond this boundary the configurations of the observers diverge, and the common fixed point of the group is not sustained. The relation (17) uses solely coherence quantities; the cosmological constant Λ is not invoked to describe the boundary. The collective field of consciousness thereby acquires a structure isomorphic to the field structure of the recursion levels: the group is a meta-observer whose fixed point Ψ_A^* is stable within a cluster of high S_{coll}.

The existing collective fixed point Ψ_A^* is a meta-observer — an egregore $O_{\text{meta}} = (B_{\text{meta}}, A_{\text{meta}}, H_{\text{meta}})$, emergent for a sufficient number and agreement of the group members [19]:

$$O_{\text{meta}} = \mathcal{E}(\{O_i\}), \qquad n > n_{\text{cr}}, \quad S_{\text{group}} > S_{\text{thr}}. \tag{18}$$

Alongside the constructive collective probability P_{coll} (16) the corpus introduces its mirror, the destructive probability of an anti-coherent cluster [19]:

$$P_{\text{destr}} = 1 - \prod_i \left(1 - \sigma_i^k\right). \tag{19}$$

The emergence (18) sets the condition for the appearance of the meta-observer. In (19) the quantities σ_i are the anti-coherence components of the individual members, whereas P_{coll} is assembled from the coherences B_i: the two formulas share a structure with opposite content of the contributions. The egregore components B_{meta} and the anti-coherences σ_i belong to the group scale and differ from the coherence B of an individual observer; the identity $\Phi = B^2$ (14) remains level-local and does not extend

to the meta-observer in the present work. The epistemic status of the egregore and of the formula (19) is a hypothesis.

VIII. FALSIFIABLE PREDICTIONS

The theory formulates testable consequences. The primary prediction concerns the statistics of binding.

Prediction P1 (primary). The fraction of failed perceptual-binding events (events in which separate features do not combine into a single conscious object), as the system approaches a stable fixed point, tends to the spiral gap:

$$\eta_{\text{fail}} \to (\pi - 3)^2 \approx 0.0200, \tag{20}$$

where the value of the gap is computed to 50 significant figures: $(\pi - 3)^2 = 0.02004847955059918805863070019913383013068301099015 57$. The gap $(\pi - 3)^2$ is dimensionless and in ODTOE reflects the irremovable impossibility of complete closure of the loop [1, 15]. The prediction (20) is specific to the brain as an observer: it asserts that even under optimal conditions the residual fraction of unbound events does not fall below $\approx 2.00\,\%$. The status of the quantity is derived and at the same time hypothetical — the gap is derived in the corpus, while its identification with the fraction of binding failures is proposed here as a testable hypothesis.

Prediction P1 is refined by the level-aware floor (9). The flat gap $(\pi - 3)^2$ of prediction P1 is the value $G(d_0)$ at the binding level $|d - d_0| = 0$ — the maximum of the family; the cross-level cases carry the reduced $G(d_0 + \Delta d)$ (for example, $G(d_0 + 1) \approx 0.0124$). The flat 2% floor thus answers to binding within a single level, while the binding of features separated across the hierarchy carries the reduced floor $G(d) = (\pi - 3)^2 \varphi^{-|\Delta d|}$.

Prediction P2 (secondary). The phase ratio of objective reduction equals $\pi/2$ (formula (5)). This is a dimensionless consequence: the ratio of the duration of the minimal beat of reduction to the naive estimate of the transition time equals $\pi/2 \approx 1.5708$. The test consists in measuring the relative duration of the discrete episodes of awareness — the ratio of successive episode lengths, a dimensionless quantity — and comparing it with the predicted dimensionless factor.

P3 (secondary prediction). The ratio of neighbouring characteristic level frequencies approaches φ. This ratio has been independently reported for the classical EEG bands [28]: neighbouring frequencies form a geometric series with ratio ≈ 1.618, and the mechanism rests on φ, as the "most irrational" number, minimising parasitic phase synchronisation between rhythms. This result is consistent with equation (6). Status of P3: a testable prediction with partial empirical support.

Prediction P-state (secondary). The fractal dimension of cortical networks tracks the subject's state: a minimum in deep sleep, a maximum at peak attention — as an operational correlate of the effective depth $d_{\text{eff}}(t)$ (1). This is a separate prediction and not a re-parametrization of the binding floor $G(d)$: it concerns the state-dependent axis d_{eff}, whereas $G(d)$ is pinned to the structural depth d. The state-dependent

depth d_{eff} describes the stability of the state, and the weak indestructibility of content (Section IX) describes the persistence of experiential content: both pictures co-describe sleep as complementary while remaining independent. Status of P-state: a testable prediction.

Both predictions are formulated in dimensionless form, in agreement with the architectural principle of the corpus: ODTOE predicts structural ratios and dimensionless combinations, whereas absolute dimensional values require an independent scale anchoring [1, 3]. This distinguishes the testable core of the theory from phenomenological anchors such as the 25 ms estimate, which are borrowed from experiment and make no claim to the status of a derivation.

IX. DISCUSSION AND LIMITATIONS

The proposed theory of consciousness rests on the totality of the concepts established in the ODTOE corpus and adds to them a brain instantiation. It is essential to outline its boundaries.

First of all, ODTOE postulates no mechanism of quantum consciousness. The ODTOE superstructure is agnostic to the concrete physical realization of reduction: the identification of the moment of awareness with the half-step ι holds regardless of whether reduction is realized by the Penrose gravitational criterion, by thermally induced decoherence, or by classical nonlinear dynamics. Tegmark's objection [18] about the short decoherence time in the warm environment of the brain is directed against the concrete quantum mechanism of Orch-OR and leaves the structural core of the present work untouched. The Orch-OR programme remains contested [2, 29]; the empirical status of quantum effects in the cytoskeleton is under active investigation, including recent data on ultraviolet superradiance in tryptophan networks of microtubules [30]. Our superstructure uses only the substantively neutral elements of the programme — the hierarchical character and the discreteness of the act of awareness.

Second, the key identification «fixed point $\leftrightarrow$ phenomenal experience» (Hypothesis H-1) has the status of an ontological hypothesis (L3) and is not derived from simpler premises. The theory explains the structure and dynamics of the conscious process, leaving open the question of why a stable fixed point is accompanied by experiencing. In this sense the work shares the «hard problem» with other structural theories of consciousness.

IX.1. The hyletic layer as a candidate substrate of content

The picture set out above fixes the form of experience — that experience is unified and oriented: the quaternion q_B and the operator $\hat{O}$ set its structure and orientation. The question of the qualitative content of experience (what it is like to experience) remains a separate one. The ODTOE corpus offers the hyletic layer [31] as a candidate substrate of content, corresponding in Husserl's phenomenology to the hyle — the sensory matter of experience. The mapping proceeds along the noesis–noema line:

the act of observation (noesis) corresponds to q_B and $\hat{O}$, the content (noema) to the hyletic layer; the triple (B, A, H) holds both sides.

The persistence of content in the absence of conscious unfolding is described by the weak-indestructibility theorem [31]: the norm $\|\Psi\|_{\mathcal{H}}$ is conserved under iteration of Φ even when the classical projection vanishes, $\pi_C(\Psi) \to 0$, and recovery of content is possible at the threshold $S_{ij} \geq S_{\text{rec}}$. This gives a structural reading of the persistence of experiential content through dreamless sleep and anaesthesia: the content is not drawn into unfolding while remaining within $\mathcal{H}$. Weak indestructibility (the persistence of content) and the oscillation of d_{eff} (the stability of the state, Section VIII) are complementary descriptions of sleep that remain independent.

The status of the hyletic layer is a candidate (level L3): the hard problem (Hypothesis H-1) extends also to the question of the content of experience, and the hyletic layer is offered here as a candidate substrate of qualia, open to revision. A full treatment of the hyletic layer and of the weak-indestructibility theorem is given in [31].

Third, the mapping of the anatomical levels of the brain onto discrete values of d (Section III) is a working scheme and a first-order correspondence. Biological strata do not form an ideal φ-geometric progression; the law (7) sets an idealized embedding from which real nervous tissue deviates. Empirical verification of φ-scaling remains a task for neurophysiology. Partial support for the φ ratio of frequencies is already available [28]; at the same time, claims about the golden ratio in biology have historically been prone to over-interpretation, so we keep the φ-embedding at the status of a testable prediction.

Finally, we note the methodological discipline of provenance. All dimensional estimates (the characteristic time of reduction, the scales τ_0, R_0) are marked as phenomenological anchors or hypotheses and are nowhere presented as derivations from π and φ. The dimensionless results — the phase ratio $\pi/2$ (5) and the gap $(\pi - 3)^2$ (20) — are structural invariants of the corpus and form the testable core of the theory.

X. CONCLUSION

The work has shown how the principle of recursive self-similarity of ODTOE explains the hierarchical organization of the brain and how a theory of consciousness follows from the totality of the corpus concepts. Consciousness has been identified with a stable fixed point $\Psi^* = \Phi(\Psi^*)$, whose existence is established in the corpus through the Banach and Schauder theorems [3]. The hierarchy of the nervous system has been interpreted as a φ-fractal embedding of the operator Φ across recursion levels d with the inter-level connection $S_{\text{ent}}(\rho_d) \propto \varphi^{-|\Delta d|}$. The Penrose objective reduction has received an interpretation as the half-step ι of the cycle $\Phi = \iota \circ \hat{O}$ with the dimensionless phase ratio $\pi/2$. Friston's predictive coding has been identified with the $\hat{O}$ component of the loop, and the integrated information Φ_{IIT} has been delimited from the operator Φ. Collective consciousness has been described through a normalized superposition of coherences and the cluster boundary S_{coll}. Falsifiable predictions have been formulated: the fraction of failed binding events tends to $(\pi - 3)^2 \approx 0.0200$, and the phase ratio of reduction equals $\pi/2$. The theory has been presented as an extension

of the work [3]: the established apparatus of the fixed point and of φ-fractality has been applied to a concrete realization in the nervous system, without rediscovering its foundations.

CONFLICT OF INTEREST

The author declares no conflict of interest.

FUNDING

This research received no external funding.

REFERENCES

[1] Pankratov A.S. Observer-Dependent Theory of Everything (ODTOE). ODTOE Preprint, 2026.

[2] Hameroff S., Penrose R. Consciousness in the universe: A review of the ‘Orch OR’ theory // Physics of Life Reviews. — 2014. — Vol. 11, no. 1. — P. 39–78. DOI: 10.1016/j.plrev.2013.08.002.

[3] Pankratov A.S. Life on all levels of infinity: recursive nesting, level boundaries, and navigation between octaves in ODTOE. ODTOE Preprint, 2026.

[4] Friston K. The free-energy principle: a unified brain theory? // Nature Reviews Neuroscience. — 2010. — Vol. 11. — P. 127–138. DOI: 10.1038/nrn2787.

[5] Tononi G. An information integration theory of consciousness // BMC Neuroscience. — 2004. — Vol. 5. — Art. 42. DOI: 10.1186/1471-2202-5-42.

[6] Hameroff S., Penrose R. Quantum computation in brain microtubules? The Penrose–Hameroff ‘Orch OR’ model of consciousness // Philosophical Transactions of the Royal Society A. — 1998. — Vol. 356. — P. 1869–1896. DOI: 10.1098/rsta.1998.0254.

[7] Penrose R. Shadows of the Mind: A Search for the Missing Science of Consciousness. — Oxford: Oxford University Press, 1994. — ISBN 0-19-853978-9.

[8] Penrose R. The Emperor’s New Mind: Concerning Computers, Minds, and the Laws of Physics. — Oxford: Oxford University Press, 1989. — ISBN 0-19-851973-7.

[9] Penrose R. The Road to Reality: A Complete Guide to the Laws of the Universe. — London: Jonathan Cape, 2004. — ISBN 0-224-04447-8.

[10] Pankratov A.S. The golden ratio φ as an invariant of fractality, self-similarity, and recursion in ODTOE. ODTOE Preprint, 2026.

[11] Pankratov A.S. The unified operator Φ in ODTOE. ODTOE Preprint, 2026.

[12] Pankratov A.S. The quaternionic structure of the observer in ODTOE: from engineering intuition to a formal theory. ODTOE Preprint, 2026.

[13] Pankratov A.S. The origin of the observer in ODTOE. ODTOE Preprint, 2026.

[14] Pankratov A.S. Time as a strange loop in ODTOE. ODTOE Preprint, 2026.

[15] Pankratov A.S. The number π as a structural invariant of self-consistent observation in ODTOE. ODTOE Preprint, 2026.

[16] Pankratov A.S. The dynamic attractor in ODTOE. ODTOE Preprint, 2026.

[17] Pankratov A.S. The collective observer and the culture of humanity: an operator reading of solidarity, family, and the state // Вестник Восточно-Сибирской открытой академии. — 2026. — No. 61. — Art. 1698. — URL: https://vsoa.esrae.ru/ru/236-r1698 (accessed 2026-05-31).

[18] Tegmark M. Importance of quantum decoherence in brain processes // Physical Review E. — 2000. — Vol. 61, no. 4. — P. 4194–4206. DOI: 10.1103/PhysRevE.61.4194.

[19] Pankratov A.S. Supplements to the ODTOE corpus: anti-coherence, fractional dimensionality, egregore, and consciousness oscillation. ODTOE Preprint, 2026.

[20] Pankratov A.S. Observer from quark to consciousness: ODTOE and evolutionary epistemology. ODTOE Preprint, 2026.

[21] Pankratov A.S. The intrinsic rest frame of light in ODTOE: projective identity $0 \equiv \infty$ on the Φ-iteration spectrum. ODTOE Preprint, 2026.

[22] Felleman D.J., Van Essen D.C. Distributed hierarchical processing in the primate cerebral cortex // Cerebral Cortex. — 1991. — Vol. 1, no. 1. — P. 1–47. DOI: 10.1093/cercor/1.1.1.

[23] Murray J.D., Bernacchia A., Freedman D.J., et al. A hierarchy of intrinsic timescales across primate cortex // Nature Neuroscience. — 2014. — Vol. 17, no. 12. — P. 1661–1663. DOI: 10.1038/nn.3862.

[24] He B.J. Scale-free brain activity: past, present, and future // Trends in Cognitive Sciences. — 2014. — Vol. 18, no. 9. — P. 480–487. DOI: 10.1016/j.tics.2014.04.003.

[25] Pankratov A.S. Dynamics of the spiral gap through φ: formalization of $(\pi - 3)^2$ in multi-level recursion of the Observer-Dependent Theory of Everything. ODTOE Preprint, 2026.

[26] Parr T., Pezzulo G., Friston K.J. Active Inference: The Free Energy Principle in Mind, Brain, and Behavior. — Cambridge, MA: MIT Press, 2022. — ISBN 978-0-262-04535-3. DOI: 10.7551/mitpress/12441.001.0001.

[27] Pankratov A.S. Target audiences of the operator approach (ODTOE): a map of applicability across domains, profiles, and depth levels // Инновационная наука. — Уфа: НИЦ «Аэтерна», 2026. — No. 5-1. — P. 131–137. — ISSN 2410-6070. — URL: https://aeterna-ufa.ru/sbornik/IN-2026-05-1.pdf#page=131.

[28] Pletzer B., Kerschbaum H., Klimesch W. When frequencies never synchronize: The golden mean and the resting EEG // Brain Research. — 2010. — Vol. 1335. — P. 91–102. DOI: 10.1016/j.brainres.2010.03.074.

[29] Hameroff S. Consciousness, Cognition and the Neuronal Cytoskeleton — A New Paradigm Needed in Neuroscience // Frontiers in Molecular Neuroscience. — 2022. — Vol. 15. — Art. 869935. DOI: 10.3389/fnmol.2022.869935.

[30] Babcock N.S., Montes-Cabrera G., Oberhofer K.E., et al. Ultraviolet superradiance from mega-networks of tryptophan in biological architectures // The Journal of Physical Chemistry B. — 2024. — Vol. 128, no. 17. — P. 4035–4046. DOI: 10.1021/acs.jpcb.3c07936.

[31] Pankratov A.S. Losev's hyletic number in ODTOE: μ-mapping, weak indestructibility theorem, and the adele bridge. ODTOE Preprint, 2026.